Note of Quantum Optics

相干态

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle.$$

湮灭算符. not Hermitian. 非可观测量. 复本征值.

相干态湮灭一个光子后, 状态不发生改变.

$$\langle n|\hat{a}|\alpha\rangle = \sqrt{n+1}\langle n+1|\alpha\rangle = \alpha\langle n|\alpha\rangle.$$

$\langle n| = \langle 0|\frac{\hat{a}^n}{\sqrt{n!}}$. 有 $\langle n|\alpha\rangle = \langle 0|\frac{\hat{a}^n}{\sqrt{n!}}|\alpha\rangle = \frac{\hat{a}^n}{\sqrt{n!}}\langle 0|\alpha\rangle.$

$$1 = \langle\alpha|\alpha\rangle = \sum_n \langle\alpha|n\rangle\langle n|\alpha\rangle = |\langle 0|\alpha\rangle|^2 \sum_n \frac{|\alpha|^{2n}}{n!} = \underbrace{|\langle 0|\alpha\rangle|^2}_{\text{取为 } e^{-|\alpha|^2}} e^{|\alpha|^2}.$$

因此 $|\alpha\rangle$ 在粒子数表象下

$$|\alpha\rangle = \exp\left(-\frac{1}{2}|\alpha|^2\right)\sum_n \frac{\alpha^n}{\sqrt{n!}}|n\rangle.$$

或写为 $|\alpha\rangle = \exp\left(-\frac{1}{2}|\alpha|^2\right)\sum_n \frac{(\alpha a^\dagger)^n}{n!}|0\rangle = \exp\left(-\frac{1}{2}|\alpha|^2\right)e^{\alpha a^\dagger}|0\rangle.$

一些基础复习

$$i\hbar\frac{d|\psi\rangle}{dt}=\hat{H}|\psi\rangle.$$

H 不含时，有形式解 $|\psi(t)\rangle=U(t)|\psi(0)\rangle=e^{-iHt/\hbar}|\psi(0)\rangle$.

$\hat{H}(t)$ 依赖于时间 t 时，形式解

$$U(t)=\hat{T}\exp\left[\frac{1}{i\hbar}\int_0^t d\tau\,\hat{H}(\tau)\right]$$

编时算符 $\hat{T}$，对展开式中算符组合按时间发生次序从大到小排列 $t\geq\tau_1\geq\tau_2\cdots$

$$=1+\frac{1}{i\hbar}\int_0^t d\tau\,\hat{H}(\tau)+\left(\frac{1}{i\hbar}\right)^2\int_0^t d\tau_1\int_0^{\tau_1}d\tau_2\,\hat{H}(\tau_1)\hat{H}(\tau_2)+\cdots$$

特别地，不同时刻的哈密顿量对易时

$$\left[\hat{H}(\tau_i),\hat{H}(\tau_j)\right]=0.$$

有 $$\int_0^t d\tau_1\int_0^{\tau_1}d\tau_2\cdots\int_0^{\tau_{n-1}}d\tau_n\,\hat{H}(\tau_1)\hat{H}(\tau_2)\cdots\hat{H}(\tau_n)=\frac{1}{n!}\left[\int_0^t d\tau\,\hat{H}(\tau)\right]^n.$$

坍缩后的重复测量

- 若本征态 $|a\rangle$ 也为 $\hat{B}$ 的本征态，不变。

- 非……，则可展开

$$|a\rangle = \sum_b \langle b|a\rangle |b\rangle.$$

可见 $\hat{A}$、$\hat{B}$ 对易时，测量后系统状态才不为后续测量改变，因此时它们有共同本征态。

用对易子刻画算符间不可对易的程度。

纯态系综的观测量平均值

$$\langle \hat{A} \rangle = \sum_n p_n \langle \psi_n|\hat{A}|\psi_n\rangle = \sum_a \sum_n p_n \langle \psi_n|\hat{A}|a\rangle\langle a|\psi_n\rangle$$

$$= \sum_a \langle a|\left(\sum_n p_n|\psi_n\rangle\langle\psi_n|\right)\hat{A}|a\rangle = \mathrm{Tr}\left[\rho\hat{A}\right].$$

Quantum Optics

probe experimentally   quiz ?

general layout   Textbook : Introductory Quantum Optics

mid-term early November   c-QED   final presentation

QM. EM.

- Photoelectric Effect
- Ultraviolet catastrophe
- Stable Orbitals

Find Canonical q and momentum p of a Hamiltonian H.

$$\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i} \quad \frac{dp_i}{dt} = -\frac{dH}{dq_i}.$$

Basic strategy / form

$$H = V(x,y,z) + \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2)$$

$$\frac{dx}{dt} = \frac{p_x}{m}. \qquad \frac{dp_x}{dt} = -\frac{\partial V}{\partial x}.$$

$$[\hat{q}_i, \hat{p}_j] = i\hbar\,\delta_{ij}.$$

Dirac Notation

Ket Vector $|\psi\rangle$

or corresponding bra $\langle\psi|$

$$|\psi\rangle = c_1|a_1\rangle + c_2|a_2\rangle + \cdots$$

$$= \sum_n c_n|a_n\rangle.$$

$\langle\phi|\psi\rangle \longrightarrow$ prob. amplitude

$$\langle a_i|a_j\rangle = 0. \quad \text{orthogonal}$$

$|\langle\phi|\psi\rangle|^2 \qquad \langle\psi|\phi\rangle^*$

Operator $\hat{O}|\psi\rangle \rightarrow |\phi\rangle$. $\langle\psi|\hat{O}^\dagger$ $\langle\phi|$

adjoint operator

$(\hat{A}\hat{B})^\dagger = \hat{B}^\dagger\hat{A}^\dagger$ $\hat{U}^\dagger\hat{U} = 1$.

Eigenvalue Equation

$$\hat{A}|a_n\rangle = a_n|a_n\rangle.$$

eigenvalues

Hermitian $\longrightarrow$ physical observables $\longrightarrow$ real systems

$\hat{A} = \hat{A}^\dagger$.

position $x \longrightarrow \hat{x}$

momentum $p_x \longrightarrow \hat{p}$

interference measurements

$|\psi\rangle \longrightarrow |a_n\rangle$ $\rightarrow$ measurement collapse

Avg $\langle\hat{A}\rangle = \langle\psi|\hat{A}|\psi\rangle$.

expectation value for operators

Uncertainty

$$\Delta A = (\langle A^2\rangle - \langle A\rangle^2)^{1/2}.$$

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}.$$

$= 0$. $\neq 0$.

Simultaneous eigenstates

$[\hat{x}, \hat{p}_x] = i\hbar$.

x, p conjugate variables

$\hookrightarrow [\hat{A}, \hat{B}] = iC$.

$$\Delta A^2 \Delta B^2 \geq \frac{|\langle C\rangle|^2}{4}.$$

Schrödinger Equation

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}|\psi\rangle.$$

Heisenburg Equation

Stationary states

$$\frac{d\hat{O}}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{O}]. \qquad \hat{H}|E\rangle = E|\hat{E}\rangle.$$

45 min

Review $\vec{E}$ electric field (V/m)

$\vec{B}$ magnetic field (T)

$\vec{D}$ displacement field $= \varepsilon_0 \vec{B} + \vec{P}$

change --- is

$\vec{P} = \varepsilon_0 \chi \vec{E}$.

$\rho_{onem} =$

$\nabla \cdot \vec{D} = \rho +$ charge density.

$\nabla \cdot \vec{B} = 0$

$\nabla \times G =$

$\nabla \times \vec{H} = \vec{D} + \frac{\partial D}{\partial t}$.

$E(\zeta, k) = E_a >$

$\rho_i = 0, j = 0$

$\frac{1}{\mu_0 \mu_r} \nabla \times \vec{B} = \varepsilon_0 \varepsilon_r \frac{\partial E}{\partial t}$

$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t}(\nabla \times \vec{B}) = -\mu_0 \mu_r \varepsilon_0 \varepsilon_r \frac{d^2 E}{dt^2}$.

$\downarrow$

$\vec{\nabla}(\underbrace{\vec{\nabla} \cdot \vec{E}}_{0}) - \nabla^2 E$

$\nabla^2 E = \mu_0 \mu_r \varepsilon_0 \varepsilon_r \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{v^2} \frac{d^2 E}{dt^2}$.

$\frac{1}{v^2} = \mu_0 \mu_r \varepsilon_0 \varepsilon_r$.

$v = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{c}{n}$.

$\vec{E}(\vec{r}, t) = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$

$\vec{B}(\vec{r}, t) = \vec{B}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$.

K is wave vector. $|\vec{k}| = \frac{2\pi}{\lambda}$. $\frac{\omega}{k} = \frac{c}{n}$.

$\vec{E}(\vec{r}, t) = Re(\vec{E}_0 e^{i\vec{k} \cdot \vec{r} - \omega t})$

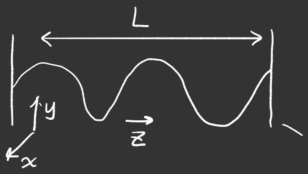
$= \vec{E}_0 \cos(\vec{k} \cdot \vec{r} - \omega t)$.

$\nabla \cdot \vec{E}(\vec{r}, t) = i\vec{k} \cdot \vec{E}(\vec{r}, t) = 0$. $\vec{E}_0 \perp \hat{k}$.

$\nabla \times \vec{E}(\vec{r}, t) = -\frac{\partial \vec{B}}{\partial t} = i\vec{k} \times \vec{E}_0 = i n \vec{B}_0$. $\vec{E}_0 \perp \vec{B}_0$.

Quantum Theory of Light → Loudon

Vector Quantum QTL 4.4 IQO 2.4



1D Cavity.

$$\vec{E}_x(z,t) = E_0 \sin kz \sin \omega t. \qquad k_m = \frac{m\pi}{L}, \quad m = 1, 2, 3 \ldots$$

$$\vec{B}_y(z,t) = B_0 \cos kz \cos \omega t.$$

$$H = \int^V \tfrac{1}{2}\left(\varepsilon_0 E^2 + \tfrac{1}{\mu_0} B^2\right) \qquad \int \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{\varepsilon_0}{2} A \int_0^L E_0^2 \sin^2 k_z \sin^2 \omega t \, dz = \tfrac{\varepsilon_0 V}{4} E_0^2 \sin^2 \omega t.$$

V is mode volume.

$$\int^V \frac{1}{2\mu_0} B^2 = \frac{V}{4\mu_0} B^2 \cos^2 \omega t$$

$q(t)$, $p(t)$

$$H = \frac{V}{4}\left(\varepsilon_0 E_0^2 \sin \omega t + \frac{B_0^2}{\mu_0} \cos^2 \omega t\right)$$

$$q(t) = \sqrt{\frac{\varepsilon_0 V}{2\omega^2}} E_0 \sin \omega t \qquad p(t) = \sqrt{\frac{V}{2m\mu}} B_0 \cos \omega t.$$

$$H = \tfrac{1}{2}(p^2 + \omega^2 q^2).$$

(1) Harmonic OSC

$$p_x = m\dot{x}, \quad \dot{p}_x = -m\omega^2 q. \qquad [\hat{x}, \hat{p}_x] = i\hbar.$$

$$H = \frac{p_x^2}{2m} + \frac{m\omega^2}{2} x^2. \qquad [\hat{q}, \hat{p}] = i\hbar. \quad \Delta q \Delta p \geq \frac{\hbar}{2}.$$

$x \longrightarrow \hat{x}$ $\quad q \longrightarrow \hat{q}.$

$p \longrightarrow \hat{p}$ $\quad p \longrightarrow \hat{p}.$

$$\hat{H} = \tfrac{1}{2}(p^2 + \omega^2 q^2)$$

$\hat{a}$: annihlation $\quad \hat{a}^\dagger$: creation

$$\hat{a} = (2\hbar\omega)^{-\frac{1}{2}}(\omega\hat{q} + i\hat{p}) \qquad \hat{a}^\dagger = (2\hbar\omega)^{-\frac{1}{2}} \omega\hat{q} - i\hat{p})$$

$$\hat{q} = \left(\frac{\hbar}{2\omega}\right)^{\frac{1}{2}} (\hat{a} + \hat{a}^\dagger). \qquad \hat{p} = -i\left(\frac{\hbar\omega}{2}\right)^{1/2} (\hat{a} - \hat{a}^\dagger).$$

$$\hat{E}_x(z,t) = \mathcal{E}_0 \sin kz (\hat{a}+\hat{a}^\dagger) \qquad \mathcal{E}_0 = \left(\frac{\hbar\omega}{\varepsilon_0 V}\right)^{\frac{1}{2}}$$

$$\hat{B}_y(z,t) = B_0 \frac{1}{i}(\hat{a}-\hat{a}^\dagger)\cos kz \qquad B_0 = \left(\frac{\mu_0}{k}\right)\left(\frac{\varepsilon_0\hbar\omega^3}{V}\right)^{\frac{1}{2}}$$

$$[\hat{a}, a^\dagger] = \frac{i}{\hbar}(\hat{p}\hat{q}-\hat{q}\hat{p}) = \frac{i}{\hbar}\underbrace{[\hat{p},\hat{q}]}_{i\hbar} = -i^2 = 1.$$

$$H = \frac{1}{2}\hat{p}^2 \ldots$$

$|n\rangle \leftarrow$ eigenstate of s.m. field with energy E_n

$$\hat{H}|n\rangle = \hbar\omega(a^\dagger a + \tfrac{1}{2})|n\rangle = E_n|n\rangle.$$

$$a^\dagger \hat{H}|n\rangle = \hbar\omega(a^\dagger a a^\dagger + \tfrac{1}{2}a^\dagger)|n\rangle$$

$$\hat{a}^\dagger\hat{a} = \hat{a}a^\dagger - 1.$$

$$= \hbar n(\hat{a}^\dagger\hat{a}\hat{a}^\dagger - \hat{a}^\dagger + \tfrac{1}{2}\hat{a}^\dagger)|n\rangle = \hbar\omega(\hat{a}^\dagger\hat{a} - \tfrac{1}{2})\hat{a}^\dagger|n\rangle = E_n\hat{a}^\dagger|n\rangle.$$

$$\underbrace{\hbar\omega(\hat{a}^\dagger\hat{a}+\tfrac{1}{2})}_{H}\underbrace{\hat{a}^\dagger|n\rangle}_{\text{Eigenstate}} = \underbrace{(E_n+\hbar\omega)}_{+\hbar\omega}\hat{a}^\dagger|n\rangle = E_{n+1}(\hat{a}^\dagger|n\rangle)$$

Creation

$$\hat{H}(\hat{a}|n\rangle) = \underbrace{(E_n-\hbar\omega)}_{E_{n-1}}\hat{a}|n\rangle$$

annihilation

$E > 0$ define ground state 0.

$$\hat{H}(\hat{a}|0\rangle) = (E_0-\hbar\omega)\hat{a}|0\rangle = 0.$$

$$\hat{H}|n\rangle = \hbar\omega(\hat{a}^\dagger\hat{a}+\tfrac{1}{2})|0\rangle = \tfrac{1}{2}\hbar\omega|0\rangle.$$

$$E|n+1\rangle = (E_n + h\omega) \longrightarrow E_1 = E_0 + \hbar\omega$$

$$\longrightarrow E_n = (n+\tfrac{1}{2})\hbar\omega \qquad n = 0,1,2\ldots$$

$$\hat{H}|n\rangle = \hbar\omega(\hat{a}^\dagger\hat{a}+\tfrac{1}{2})|\hat{n}\rangle = \hbar\omega(n+\tfrac{1}{2})|n\rangle$$

$$a^\dagger a|n\rangle = n|n\rangle$$

number operator

$$\langle n|n\rangle = 1.$$

$$\hat{a}|n\rangle = C_n|n-1\rangle.$$

$$\langle n|\hat{a}^{\dagger}a|n\rangle = n = \langle n-1|C_n^* C_n|n-1\rangle = |C_n^2|.$$

$$C_n = \sqrt{n}, \quad \hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle.$$

$$\hat{a}^{\dagger}|n\rangle = \sqrt{n+1}\,|n+1\rangle.$$

$$|n\rangle = \frac{(a^{+})^n}{\sqrt{n!}}|0\rangle. \qquad \langle n'|n\rangle = \delta_{nn'}.$$

$$\langle n-1|\hat{a}|n\rangle = \sqrt{n}$$

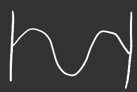
$$\langle n+1|\hat{a}^{+}|n\rangle = \sqrt{n+1}.$$

Time independence a

$$\frac{\partial \hat{O}}{\partial t} = \frac{i}{\hbar}[\hat{H},\hat{O}].$$

EM properties of this quantized field

$$\hat{E} = \mathcal{E}_0 \sin(kz)(a^{\dagger}+a).$$



$$\langle n|E_x(z,t)|n\rangle = 0\ ?$$

energy is here, but expectation value of field is 0 $\Rightarrow$ phase completely undefined!

$$\Delta_x(z,t) = \sqrt{2}\,\mathcal{E}_0 \sin(kz)\left(x+\tfrac{1}{2}\right)^{\frac{1}{2}}.$$

make it little less random?

$$|\phi\rangle \qquad \Delta n\,\Delta\phi > 1.$$

QTL. Chapter 2.

Quadratures of field

$$E_x(t) = \mathcal{E}_0(\hat{a}\,e^{-i\omega t} + \hat{a}^{\dagger}e^{i\omega t})\sin kz.$$

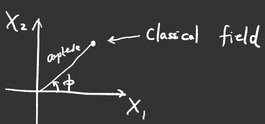
$$\hat{X}_1 = \tfrac{1}{2}(\hat{a}+\hat{a}^{\dagger}). \quad \hat{X}_2 = \tfrac{1}{2i}(\hat{a}-\hat{a}^{\dagger}).$$

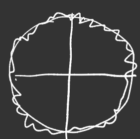
$$\hat{a}^{\dagger} = \hat{X}_1 - i\hat{X}_2. \quad \hat{a}_2^{\dagger} = \hat{X}_1 + i\hat{X}_2.$$

$$\hat{E} = \mathcal{E}_0(\hat{X}_1\cos\omega t + \hat{X}_2\sin\omega t).$$

$\hat{X}_1, \hat{X}_2$ are the amplitudes

phase at 90°





Photon counting statistics

can't directly measure the electric field of ... photons

analyze the EM field

1. Probing fluctuations & intensity over time

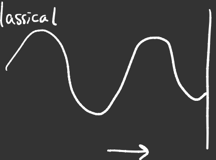
   $\bar{n}$ over time t, what is Δn?

2. Probe phase of field with temporal correlation? g_1

3. Probe temporal correlation in intensity. g_2

Coherent Light

Classical



photon detector

Constant Φ

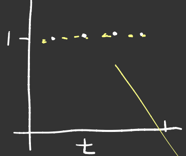
single frequency

perfectly defined phase $\longrightarrow$ energy perfectly defined ω

$\Delta x \Delta p \geq \hbar/2$

$\Delta E \Delta t \geq \hbar/2$

click



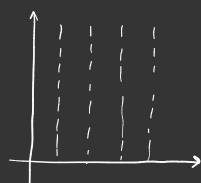
completely random events

arrival time of events

photons delocalized

how to prove this experimentally?

Statistical distribution?



N, $N >> n$.

t_N. $\frac{t}{N}$.

N sufficiently large

Chance for 1 photon $p = \frac{\bar{n}}{N}$.

0 photon $1 - \frac{\bar{n}}{N}$.

$p = \frac{\bar{n}}{N}$.

$P(n)$ prob of finding n photons within time t containing N bins.

Binomial distribution

$$P(n) = \frac{N!}{(N-n)!\, n!} p^n (1-p)^{N-n}.$$

Simplify. $N \to \infty$.

$$\lim_{N\to\infty} \rho(n) = \frac{\bar{n}^n}{n!} e^{-n}$$ Possion distribution

$$\sum_{n=0}^{\infty} n\rho(n) = \bar{n}.$$ avg. $$\sum_{n=0}^{\infty} \rho(n) = 1.$$

Δn^2. $\rho(n) = \frac{\bar{n}}{n} \cdot \rho(n-1)$ $\Rightarrow$ $\rho(n) > \rho(n-1)$. if $n < \bar{n}$.

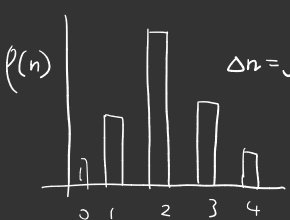
$\rho(n) < \rho(n-1)$ if $n \geq \bar{n}$.

$$\Delta n^2 = \sum_{n=0}^{\infty} (n-\bar{n})^2 \rho(n) = \sum_{n=0}^{\infty} n^2 \rho(n) - 2\bar{n} \underbrace{\sum n\rho(n)}_{\bar{n}} + n^2 \sum \rho(n).$$

$$= \frac{\sum n^2 \rho(n)}{\bar{n}^2 + \bar{n}} - \bar{n}^2 = \bar{n}.$$

$\Rightarrow \Delta n = \sqrt{\bar{n}}$. Poission Statistics

"Shot noise".



$\Delta n = \sqrt{\bar{n}}$.

measure the field of laser.

Start from well defined number of states:

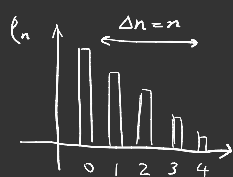
$\Delta n > \sqrt{\bar{n}}$. Super poission. ↑ Classically

$\Delta n = \sqrt{\bar{n}}$. Poission

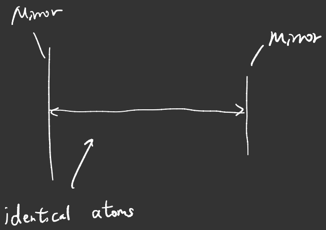
$\Delta n < \sqrt{\bar{n}}$. Sub-poission ↓ QM

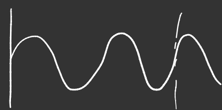
$E_n = \hbar\omega(n+\frac{1}{2})$.

$$\rho_n = \frac{e^{-E_n/k_BT}}{\sum_n e^{-E_n/k_BT}}.$$ ↖ Themal dst



T conditions → how long does it stay in phase?

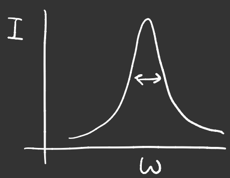




Temporal Coherence

Chapters 5, 6

or QOAI.

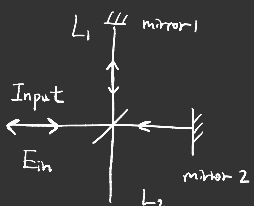


1st order coherence function

$$g'(\tau) = \frac{\langle E^*(t)E(t+\tau)\rangle}{\langle (E(t))^2\rangle}$$ ← avg intensity

value of 0 for correlation

Interferometer



$\vec{E}_1$ = components from arm 1 transmit to the output

$= \frac{E_{in}}{2} e^{i2kL_1} e^{\Delta\phi_1}$

$\vec{E}_2$ = " " $= \frac{E_{in}}{2} e^{i2kL_2} e^{\Delta\phi_2}$

$$\vec{E}_{out} = \vec{E}_1 + \vec{E}_2 = \frac{e^{i2kL_1}}{2}\left(\vec{E}_{in} - \vec{E}_{in} e^{2ik(L_1-L_2)}\right).$$

get interfered with each other

intensity of EM field — absolute value of field
related

$L_2 - L_1 = \Delta L.$

$\Delta L = \Delta t \cdot c.$

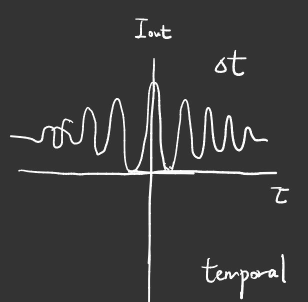
$= \tau \cdot c.$

Intensity $\sim |E|^2.$

$$E_{out} = \frac{e^{2ikL_1}}{2}\left(E_{in} - E_{in} e^{2ikL/t}\right)$$

$I(\tau) \propto E_{out}$ " t "

$I(\tau) = I_0(1 - Re[g'(\tau)]).$



temporal coherence

perfectly coherent field

will it decay?

intensity interference

Coherent States and Squeezing   Ch 3.7 IQO

$$\hat{E}_x(r,t) = i\left(\frac{\hbar\omega}{2\varepsilon_0 V}\right)^{\frac{1}{2}}\left[\hat{a}e^{i(kr-\omega t)} - \hat{a}^{\dagger} e^{-i(kr-\omega t)}\right].$$

$\langle n|\hat{E}|n\rangle = 0.$ no defined phase.

statistical poission distribution in photon counts

$\langle \psi|\hat{E}|\psi\rangle \leftarrow |n\rangle.$

$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle.$ coherent state.   can be complex.

$\langle\alpha|a^{\dagger} = a^{*}\langle\alpha|.$ (α is complex).

$|\alpha\rangle = \sum_{n=0}^{\infty} C_n|n\rangle.$ $\hat{a}|n\rangle = \sqrt{n}|n-1\rangle.$

$$\hat{a}|\alpha\rangle = \sum_{n=1}^{\infty} C_n\sqrt{n}\,|n-1\rangle = \alpha\sum_{n=0}^{\infty} C_n|n\rangle \leftarrow \text{initial state}$$

$$= \alpha\sum_{n=1}^{\infty} C_{n-1}|n-1\rangle.$$

$$C_n\sqrt{n} = \alpha C_{n-1}.$$

$$C_n = \frac{\alpha}{\sqrt{n}}C_{n-1} = \frac{\alpha^2}{\sqrt{n(n-1)}}C_{n-2}.$$

$$C_{n-1}\sqrt{n-1} = \alpha C_{n-2}.$$

$$|\alpha\rangle = C_0\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle.$$

$$\langle\alpha|\alpha\rangle = 1 = |C_0|^2\sum_n\sum_{n'}\frac{\alpha^{*n}\alpha^{n'}}{n!n'!}\langle n|n'\rangle$$

$$= |C_0|^2\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{n!} = |C_0|^2 e^{|\alpha|^2}.$$

$$|\alpha\rangle = \left(e^{-\frac{1}{2}|\alpha|^2}\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle.$$

$$C_0 = e^{-\frac{1}{2}|\alpha|^2}.$$

$\langle\beta|\alpha\rangle$

$$\langle \beta | \alpha \rangle = e^{-\frac{1}{2}|\alpha|^2 - \frac{1}{2}|\beta|^2} \sum_{n=0}^{\infty} \sum_{n=0}^{\infty} \frac{\beta^{*m} \alpha^n}{\sqrt{m!\, n!}} \langle n | m \rangle$$

$$= e^{-\frac{1}{2}|\beta - \alpha|^2} e^{\frac{1}{2}(\beta^* \alpha - \alpha^* \beta)}.$$

$$\langle \alpha | \hat{E}_x(r,t) | \alpha \rangle \qquad \hat{E}_x(r,t) = i\left(\frac{\hbar\omega}{2\varepsilon_0 V}\right)^{\frac{1}{2}} \left[\hat{a} e^{ikr-\omega t} - a^{\dagger} e^{i(kr-\omega t)}\right].$$

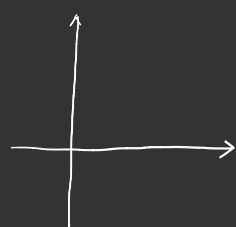
$$\alpha = |\alpha| e^{i\theta}.$$

$$= 2|\alpha| \left(\frac{\hbar\omega}{2\varepsilon_0 V}\right)^{\frac{1}{2}} \sin(\omega t - kr - \theta) \quad \leftarrow \text{classical field almost}$$

$$\langle \alpha | \hat{E}_x^2 | \alpha \rangle = \frac{\hbar\omega}{2\varepsilon_0 V}\left(1 + 4|\alpha|^2 \sin^2(\omega t - kr - \theta)\right).$$

$$\langle \Delta E_x^2 \rangle = \langle E_x^2 \rangle - \langle E_x \rangle^2 = \left(\frac{\hbar\omega}{2\varepsilon_0 V}\right)^{\frac{1}{2}} \leftarrow$$ identical to fluctuation of vacuum state

$\hat{X}_1,\ \hat{X}_2.$



$$\hat{X}_1 = \frac{1}{2}(\hat{a} + \hat{a}^{\dagger}). \qquad \hat{X}_2 = \frac{1}{2i}(\hat{a} - \hat{a}^{\dagger}).$$

$$\langle \hat{X}_1 \rangle = \langle \alpha | \frac{1}{2}(\hat{a} + \hat{a}^{\dagger}) | \alpha \rangle$$

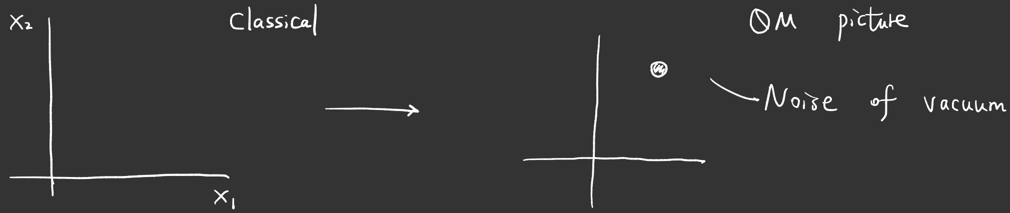
$$= \frac{1}{2}(\alpha^* + \alpha) = \mathrm{Re}[\alpha].$$

$$= |\alpha| \cos\theta. \qquad \alpha = |\alpha| e^{i\theta}.$$

$$\langle \hat{X}_2 \rangle = \mathrm{Im}(\alpha) = |\alpha| \sin\theta.$$

$$[\hat{X}_1, \hat{X}_2] = \frac{i}{2}. \quad \Rightarrow \langle \Delta \hat{X}_1 \rangle^2 \langle \Delta X_2 \rangle^2 \geq \frac{1}{16}.$$

$$\langle \Delta X_1 \rangle^2 = \langle \Delta X_2 \rangle^2 = \frac{1}{4}. \qquad \Delta X_1^2\, \Delta X_2^2 = \frac{1}{16}.$$



$$\bar{n} = \langle \alpha | \hat{n} | \alpha \rangle . \qquad \hat{n} = \hat{a}^{+} \hat{a} .$$

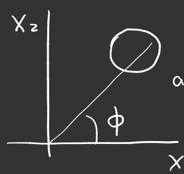
$$\bar{n} = |\alpha|^2 = \text{average photon number.} \qquad [\hat{a}, \hat{a}^{+}] = 1.$$

$$\langle \alpha | \hat{n}^2 | \alpha \rangle \quad \langle \alpha | \hat{a}^{+} \hat{a} \hat{a}^{+} \hat{a} | \alpha \rangle \qquad \hat{a} \hat{a}^{+} \hat{a} = (1 + \hat{a}^{+} \hat{a}) \hat{a} .$$

$$= \langle \alpha | \hat{a}^{+} \hat{a} + \hat{a}^{+} \hat{a}^{+} \hat{a} \hat{a} | \alpha \rangle$$

$$= \bar{n} + \bar{n}^2 = |\alpha|^2 + |\alpha|^4 .$$

$$\Delta n = |\alpha| = (\bar{n})^{\frac{1}{2}} . \leftarrow \text{Poission Statistics}$$



amplitude = $\langle \hat{n} \rangle^2$?

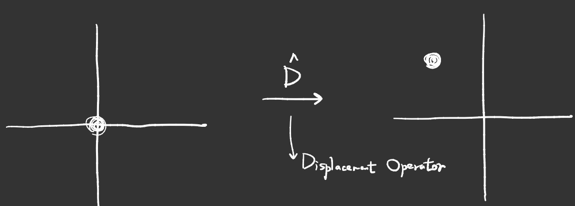
$$\Delta X_1 \, \Delta X_2 = \frac{1}{2}$$

$$|\alpha| = (\bar{n})^{\frac{1}{2}} .$$

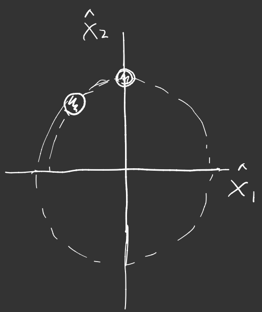
$$\Delta n = \sqrt{\bar{n}} .$$

$$\langle \phi \rangle = \theta .$$

$$\Delta \phi = \frac{\text{Arctang}\phi}{\text{Radius}} = \frac{1}{2|\alpha|} = \frac{1}{2 n^{\frac{1}{2}}} .$$



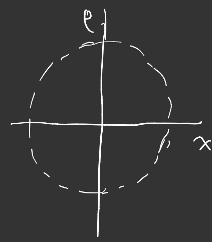
$$\hat{D}(\alpha) = e^{(\hat{a} a^{+} - \hat{a}^{*} \hat{a})} . \qquad |\alpha\rangle = \hat{D}(\alpha) |0\rangle .$$



$$|\alpha(t)\rangle = e^{iHt/\hbar}|\alpha\rangle. \qquad H = \hbar\omega\left(n+\tfrac{1}{2}\right).$$

$$= e^{-i\omega t/2}\, e^{-i\omega t/2}|\alpha\rangle$$

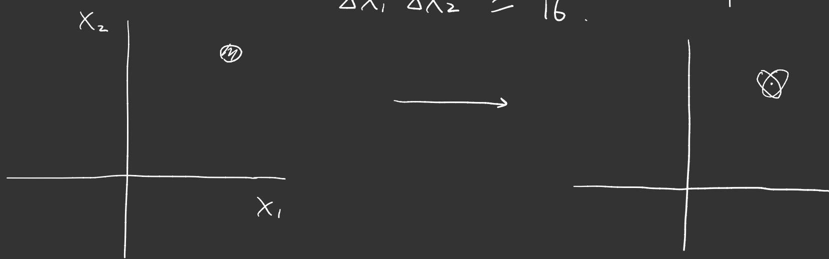
$$= e^{-i\omega t/2}|\alpha e^{i\omega t}\rangle.$$



↖ coherent state shift in phase

$$\Delta X_1^2 \Delta X_2^2 \geq \frac{1}{16}.$$

squeeze in phase



$$\hat{\xi}(\xi)\, e^{-\frac{1}{2}(\xi^*\hat{a} - \xi\hat{a}^\dagger)} \Rightarrow$$ create or destroy

2 photon at once

Square Operators

$$|\xi\rangle = \hat{\xi}|\xi\rangle|0\rangle.$$

$\xi = re^{i\theta}$ ← phase

↑ amplitude

$$\hat{\xi}^\dagger(\xi)\,\hat{a}\,\hat{\xi}(\xi) = \hat{a}\cosh r - \hat{a}^\dagger e^{i\theta}\sinh r.$$

$$\hat{\xi}^\dagger(\xi)\,\hat{a}^\dagger\,\hat{\xi}(\xi) = \hat{a}^\dagger\cosh r - \hat{a}^\dagger e^{-i\theta}\sinh r.$$

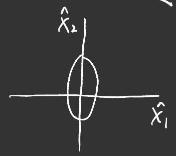
$$\langle(\Delta\hat{X}_1)^2\rangle = \frac{1}{2}\left(\cosh^2 r + \sinh^2 r - 2\sinh r\cosh r\cos\theta\right).$$

$$\langle(\Delta\hat{X}_2)^2\rangle = \frac{1}{2}\left(\cosh^2 r + \sinh^2 r + 2\sinh r\cosh r\cos\theta\right).$$

$$\theta = 0$$

$$\langle (\Delta \hat{X}_1)^2 \rangle = \frac{1}{2} e^{-2r} \qquad \langle (\Delta X_2)^2 \rangle = \frac{1}{4} e^{2r}.$$

b e c b



Squeezed Light

drive with $\vec{E} = \vec{E}_0 \cos\omega t$.

1. optical response of material

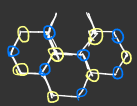
① Linear polarizability

② Non-linear polarizability

$\vec{P} =$

all material have $\chi^{(3)}$.

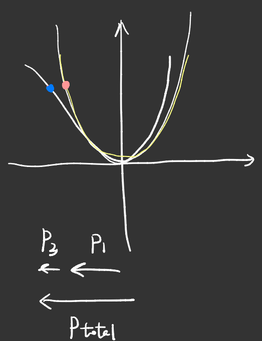
origin of $\chi^{(2)}$ non-linearity

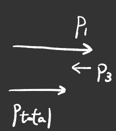


$\chi^{(2)}$ 12 × larger than $\chi^{(3)}$

inversion symmetry

create asymmetric potential for electron gas





increase field

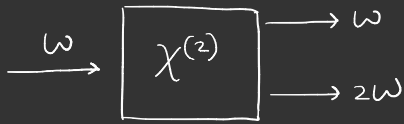
start to see

the --

DC offset --

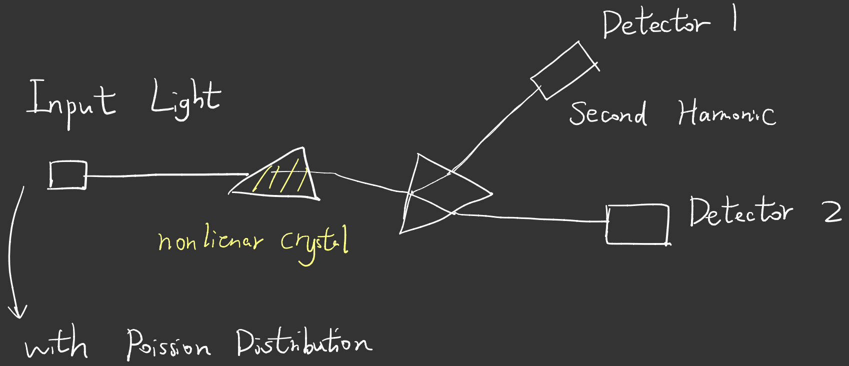
ω ... produce additional field at 2ω.

with strength $\propto E^2$



$$\omega \rightarrow \boxed{\chi^{(2)}} \rightarrow \omega,\ 2\omega$$

Second Harmonic Generation (SHG)



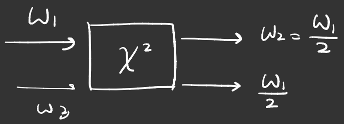
How about the distribution of second harmonic?

Sub-Poisson Distribution in SHG

Sum and -- frequency generation

small E_1 amplified by large E_2. (lock-in)

parametric amplification



(Vacuum)

⇓

no input field, still field coming out at ω_2.

enough photon pair.

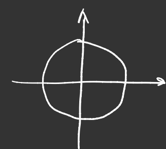
饱和 ···?

structure in material ---

periodic poling, maintain phase-matching.

thin crystal. phase match ✓.

Quadrature of the Vacuum



Squeezed vacuum.

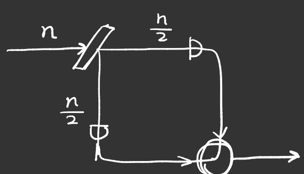
Squeezed vacuum state

How do you generate squeezing?

Hamiltonian for PDC: $H = \hbar\left[\chi^{(2)} E^* \hat{a}^2 + \chi^{(2)} E \hat{a}^{\dagger 2}\right]$.

$|t\rangle = \exp(-iHt)|0\rangle$.

Poisson statistics of split beam



Balanced Homodyne Detection

Ring Laser

unbalanced cavity

sweep the phase of LO

Noise below shot noise level for $\theta + \frac{n\pi}{2}$.

amplified noise for $\theta + n\pi$.

Limits in phase measurement in interferometry

Go to a minimum, $P_{out} \rightarrow 0$.

$$N_{out} = N_{in} \cdot \cos^2 \frac{\phi}{2}.$$

Measure arbitrary small changes in intensity:

$\Delta N_{in} = \sqrt{N_{in}}$. $\Delta N_{out} = \sqrt{N_{out}}$.

$\Delta \phi \propto \frac{1}{\sqrt{N}}$. $\frac{\Delta N_{out}}{\Delta \phi} = \frac{N_{in}}{2} \sin \phi$.

LIGO (Laser Interferometer Gravitational-Wave Observatory)

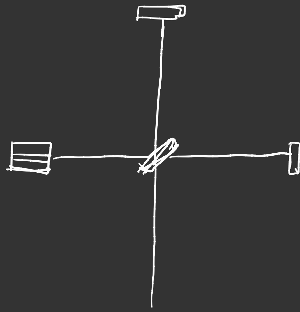
Increase N. (LIGO circulates 700 kW of laser power)

radiation pressure might shift the mirror.

Why not just increase laser power?

isolator ▯ ... confusing.

$$Var(\Delta L) \approx \frac{1}{4} \cdot \frac{\lambda^2}{4\pi^2 \alpha^3} \cdots$$



With squeezed vacuum

$$\Delta \phi = N^{\frac{1}{2}} e^{-s/2}.$$

move sensitivity below shot-noise limit.

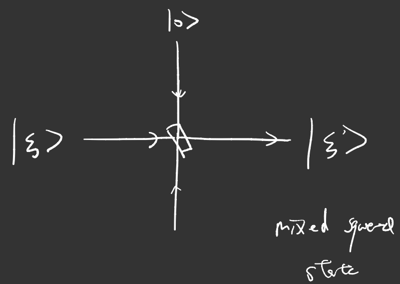
squeezing: incredibly sensitive to loss (detector inefficiency, imperfect reflectivity, imperfect mode matching etc....)

Why the impact on sensitivity ?

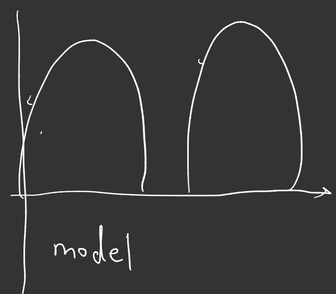
optical losses : modeled as beam splitter with T & L.

transmission loss

Where ?



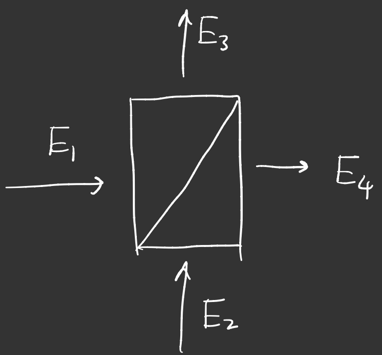
Ultimate limit of squeezed light metrology ?



reduce phase noise in measurement by reducing noise in the vacuum.

Photon Interference

– Beam splitter. Hom— – –> Q.M. of a beam splitter



$$\vec{E}_3 = R_{31}\vec{E}_1 + T_{32}\vec{E}_2.$$

complex

$$\vec{E}_4 = T_{41}\vec{E}_1 + R_{42}\vec{E}_2.$$

$$\begin{bmatrix} E_3 \\ E_4 \end{bmatrix} = \begin{bmatrix} R_{31} & T_{32} \\ T_{41} & R_{42} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}.$$

$\downarrow$

lossless BMS. Unitary

$$\begin{pmatrix} R & T \\ T & R \end{pmatrix} \begin{pmatrix} T^* & R^* \\ R^* & T^* \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

$$|E_3|^2 + |E_4|^2 = |E_1|^2 + |E_2|^2.$$

$\downarrow$

$$|R_{31}|^2 + |T_{41}|^2 = |R_{42}|^2 + |T_{41}|^2 = 1.$$

$$\phi_{31} = \phi_{42} = \phi_R.$$

$$\phi_{32} = \phi_{41} = \phi_T.$$

$$R_{31}T_{32}^* + T_{41}R_{42}^* = 0.$$

$$R_{31} = |R_{21}|e^{i\phi_{31}}$$

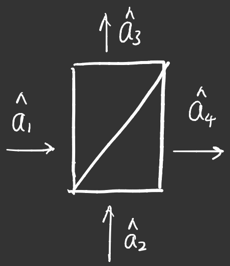
$$\phi_{31} + \phi_{42} - \phi_{32} - \phi_{41} = \pm\pi.$$

$$R_{31} = R_{42}$$

$$T_{32} = T_{41}$$

$$\phi_R - \phi_T = \pm\pi/2.$$

$\phi_R = \pi/2.$ $R_{31} = i|R_{\cdot\cdot}|.$ $|R_{31}|^2 = -|R_{\cdot\cdot}|.$



$$\hat{a}_3 = R\hat{a}_1 + T\hat{a}_2 .$$

$$\hat{a}_4 = T\hat{a}_1 + R\hat{a}_2 .$$

$$\hat{a}_3^\dagger = R^*\hat{a}_1^\dagger + T^*\hat{a}_2^\dagger .$$

$$\hat{a}_4^\dagger = T^*\hat{a}_1 + R^*\hat{a}_1^\dagger .$$

Commutation relationship

$$[\hat{a}_1, \hat{a}_1^\dagger] = [\hat{a}_2, \hat{a}_2^\dagger] = 1 . \qquad [\hat{a}_1, \hat{a}_2^\dagger] = [\hat{a}_2, \hat{a}_1^\dagger] = 0 .$$

$$[\hat{a}_3, \hat{a}_3^\dagger] = [R\hat{a}_1 + T\hat{a}_2 , R^*\hat{a}_1^\dagger + T^*\hat{a}_2^\dagger]$$

$$= RT^* + T^*R = 0 .$$

50/50 Beam Splitter, $\phi_R = \pi/2$. $\phi = 0$.

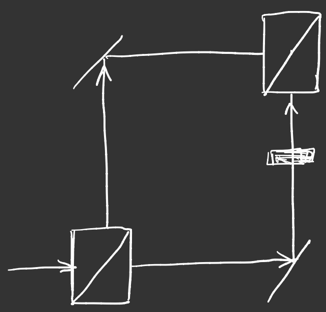
$$\hat{a}_3 = \frac{1}{\sqrt{2}}(i\hat{a}_1 + \hat{a}_2) . \qquad \hat{a}_3^\dagger = \frac{1}{\sqrt{2}}(-i\hat{a}_1 + i\hat{a}_2) .$$

$$\hat{a}_4 = \frac{1}{\sqrt{2}}(\hat{a}_4 + i\hat{a}_2) . \qquad a_4^\dagger = \frac{1}{\sqrt{2}}(\hat{a}_1^\dagger - i\hat{a}_2^\dagger) .$$

$$|1\rangle_1 |0\rangle_2 = |a_1^\dagger\rangle |0\rangle_1 |0\rangle_2 .$$

$$\hat{a}_1^\dagger \longrightarrow R\hat{a}_2^\dagger + T\hat{a}_4^\dagger = \frac{1}{\sqrt{2}}(i\hat{a}_3^\dagger + \hat{a}_4^\dagger)|0\rangle_3 |0\rangle_4$$

$$= \frac{1}{\sqrt{2}}(i|1\rangle_3 |0\rangle_4 + |0\rangle_3 |1\rangle_4) .$$



$$\hat{a}_5 = \frac{1}{\sqrt{2}}\left(i\hat{a}_4^{+} + i\hat{a}_3^{+}\right)$$

$$\hat{a}_6^{+} = \frac{1}{\sqrt{2}}\left(\hat{a}_7 + i\hat{a}_8\right)$$

$$\hat{a}_3^{+} = \hat{a}_5^{+}$$

$$\hat{a}_6^{+} = \hat{a}_4^{+} e^{i\theta}.$$

$$|\psi\rangle_{out} = e^{-i\theta/2}\left(i\sin\theta/2\, |1\rangle_7 |0\rangle_7 + \cos\frac{\theta}{2} |0\rangle_7 |1\rangle_7\right)$$

Oscillating Probabilty

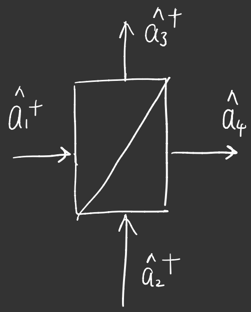
Input Coherent State

$$|\alpha\rangle_1 |0\rangle_2 = \hat{D}_1(\alpha)|0\rangle_1 |0\rangle_2 \qquad \hat{D}_1(\alpha) = e^{(\alpha a_1^{+} - \alpha^* a_1)}.$$

$$|\alpha\rangle_1 |0\rangle_2 \xrightarrow{R_3} e^{\left(\frac{\alpha}{\sigma_2}(i\hat{a}_2^{+} + \hat{a}_4^{+}) - \frac{\alpha^*}{\sigma_2}(-i\hat{a}_3 + \hat{a}_4)\right)} |0\rangle_3 |0\rangle_4$$

$$= e^{\left(\frac{i\alpha}{\sqrt{2}} a_3 + \frac{i}{\sqrt{2}}\hat{a}_3\right) + \frac{\alpha}{\sqrt{2}}(\hat{a}_4^{+} - \frac{\alpha^*}{\sqrt{2}}\hat{a}_4^{+})} |0_3\rangle |0\rangle_4$$

$$= \left|\frac{i\alpha}{\sqrt{2}}\right\rangle_3 \left|\frac{i\alpha}{\sqrt{2}}\right\rangle_4$$



$$|1\rangle_1 |1\rangle_2 = \hat{a}_1^\dagger \hat{a}_2^\dagger |0\rangle_1 |0\rangle_2$$

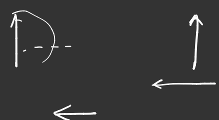
$$\xrightarrow{BS} \frac{1}{2}\left(i\hat{a}_3^\dagger + \hat{a}_4\right)\left(\hat{a}_3 + i\hat{a}_4\right)|0\rangle_3 |0\rangle_4$$

$$= \frac{1}{2}\left(i\hat{a}_3^{\dagger 2} + i\hat{a}_4^{\dagger 2} - \hat{a}_3^\dagger \cancel{\hat{a}_4^\dagger} + a_2^\dagger \cancel{a_4^\dagger}\right)|0\rangle|0\rangle$$

$$\frac{i\sqrt{2}}{2}\left(|2\rangle_3 |0\rangle_4 + |0\rangle_3 |2\rangle_4\right)$$

How does -- isolator work?

LIGO



bring back to vertical

change

Quantum state tomography and photon interference

1. One photon : Generation and Wigner Tomography

Quantum Process

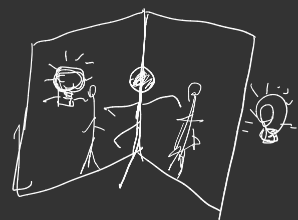
Quantum Tomography

$$\rho^{in} \longrightarrow \boxed{E} \longrightarrow \rho^{out} = E(\rho^{in})$$

Map out the process

The Wigner function $\longrightarrow$ Quantum version of a space distribution

Momentum & position. In QO ?



And map this in phase (quadrature) space

Gives a quasi-probability distribution

build up 2D images from 1D slices

Take projection along an angle in phase space

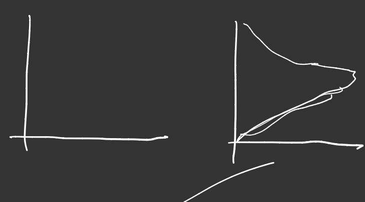
$$Pr(X_\theta) = \langle X_\theta | \hat{\rho}_{meas} | X_\theta \rangle$$

$$= \int_{-\infty}^{+\infty} W(X\cos\theta - P\sin\theta) \cdots$$

Wigner tomography. homodyne detection

$$\omega_{LO} = \omega_x$$

$$i_1 - i_2 \propto 2E_{LO}(\cos\phi_{LO} E^{X_1} + \sin\phi_{LO} E^{X_2})$$



density matrix ρ

Initially : squeezed light

Wigner function of a Fock state

$$W_n = \frac{2}{\pi}(-1)^n \cdots$$

SPDC. From pairs, how can you create a single photon Fock state?

Paper: Experimental Realization of a Localized One-Photon state.

Effect of imperfect efficiency on Fock state

$$\hat{\rho}_{meas} = \eta |0\rangle\langle 0| + (1-\eta)|1\rangle\langle 1|.$$

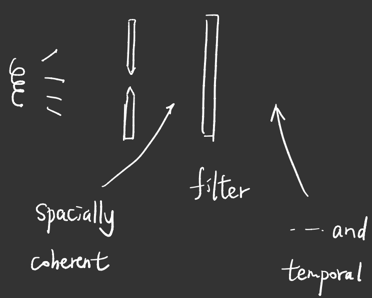
efficiency : mix vacuum with single photon state ...

Completely phase random.

$I_1 - I_2 = E_1 E_2 \cdots$

amplify --

Hong-Ou-Mandel measurement



5×10^{12} Hz. (pass bands of filter)

coherence time 100 fs.

useful for generating entanglement

No coincidences when photon overlap within coherence time. (identical photons)

One photon: Generation and Wigner tomography

Two photon: Beam splitter and ...

Four photon:

Quantum Phase Shifter

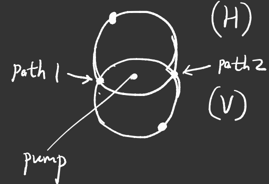
$\hat{U}_{ph} = e^{(-i a a^{\dagger} \phi)}$ $\qquad$ $\hat{U}^{\dagger}_{ph}\, a\, \hat{U}_{ph} = a e^{-i\phi}$

$|\alpha\rangle \longrightarrow \hat{U}_{ph}\, \hat{a}$ ---

proportional to number of photons

$|n\rangle \longrightarrow \hat{U}_{ph} |n\rangle = e^{-in\phi} |n\rangle$ $\qquad$ phase rotation $n\phi$ on Fock state!

Fock states pick up phase n times faster



$|H_1\rangle|V_2\rangle + |V_1\rangle|H_2\rangle$ $\qquad$ Bell state

entangled in polarization --- $\qquad$ teleportation

Super-phase resolution

1 plse~
2 · ~
4 - -

Potential application:

Quantum Lithography $\qquad$ Quantum Metrology

Bell's inequalities and teleportation

Quantum Entanglement in QO

Q communication   Q computation   Distributed Quantum Sensing

1. Rigging a Quantum game with Bell states

$$\text{Alice} \quad \xleftarrow{x=0 \text{ or } 1} \quad \xrightarrow[\;]{a=0 \text{ or } 1} \quad \text{baby} \quad \xrightarrow{y=0 \text{ or } 1} \quad \xleftarrow[\;]{b=0 \text{ or } 1} \quad \text{Bob}$$

if $xy=0$. AB win if $a+b=0$. or $a+b=2$. baby win $a+b=1$

$xy=1$. $=1$. $=0$ or 2.

How often can A and B win?

$\nearrow$ 75%.

x	y
0	0
0	1
1	0
1	1

what if baby learn game theory?

2. EPR

locality realism

On the EPR paradox

Bell

Bell's theorem — a simple proof

3 properties. A. B. C

Chart of probability

$P_{same}(B, C) >$

$\geq 1.$

start with two photon Bell state

A $\quad |a_1\rangle = |H\rangle$

$|a_2\rangle > V_2,$

correlation

... still bell state in entagled come

what bases we measure. get completely random --

maximum entangled states.

How Bell states violates ---

write the wavefunction in a,b -- basis

non-local, local

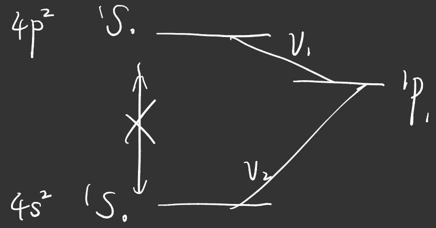
Change --- in -- polarization degrees?

How to test Bell -- inequality?

allowed range of values for variable S.

Bell state from cascaded emission

CHSH inequality



g_2 ---

$$\psi(\nu_1, \nu_2) = \frac{1}{\sqrt{2}}\left(|\sigma_+ \sigma_+\rangle + |\sigma_- \sigma_-\rangle\right).$$

Creates supersition of circularly polarized photons propagating in opposte directions

distinct signatures of polarization correlation

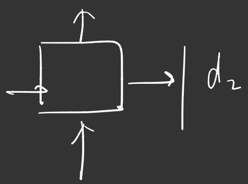
momentum contribution? --

Still many doubters.

1. half of light is thrown out

2. Calcium knows about polarizers before they emitted light

strong signature of correlation



exit at the same port?

$|a\rangle$ incident on port 1

$|b\rangle$ incident on port 2

$$\left[\begin{array}{ll} |a_3\rangle|b\rangle_3 & RT \\ |a_4\rangle|b\rangle_4 & TR \\ |a_3\rangle|b\rangle_4 + |a_4\rangle|b\rangle_3 & \dot{R}\dot{R}+\dot{T}\dot{T}=0. \\ |a_3\rangle|b\rangle_4 - |a_4\rangle|b\rangle_3 & RR-TT=1. \end{array}\right.$$

Why we cannot actually have this wavefunction?

anti-symmetric

Dof we are leaving out here?

time domain... Polarization!

total symmetry of function

asymmetric polar... & spin

total - sym... & exchange

$|a_V\rangle|b_V\rangle$

$|a_H\rangle|b_V\rangle - |a_V\rangle|b_H\rangle$ Antisymmetric polarization

Ψ $|A_{sym}\rangle_{spatial} \times |A_{sym}\rangle_{Polarization}$ conditional

$= \frac{1}{2}\left(|A_{3H}\rangle|b_{4V}\rangle - |a_{3V}\rangle|b_{4H}\rangle - |a_{4H}\rangle|b_{3V}\rangle + |a_{4V}\rangle|b_{3H}\rangle\right)$ both entangled

$= \frac{1}{\sqrt{2}}\left(|H\rangle_3|V\rangle_4 - |V_3\rangle|H_4\rangle\right) = \frac{1}{\sqrt{2}}\left(a \quad - a_{1V}^\dagger a_{2H}^\dagger\right)|0_{1H}\rangle|0_{1V}\rangle|0_{2H}\rangle|0_{2V}\rangle$

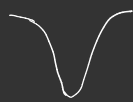
Experimental Scheme

If coincidence detected at ---, state has been teleported from 1 to 3.

Do entangle process. --- 50% correct state

dip in correlation measurement

+45° teleportation. -45° --- .



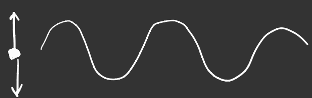
High fidelity teleportation from state 1 to 3.

Second Part: Quantum Properties of light --- interacting with matter

Next Tuesday midterm. 8:00 am. --- no class.

Light-matter Interaction QTL. Chapter 8.9.

Classical: how to generate light in a medium?



oscillating charge create light

QM description $\longrightarrow |\psi\rangle$. $\psi(x,t) = \phi(x)\, e^{-iHt/\hbar}$.

$H\psi = i\hbar \frac{\partial \psi}{\partial t}$. $\psi^*\psi = \phi^*\phi$.

what can give probability oscillates in time?

$$\psi(x,t) = c_1\phi_1\, e^{-iE_1t/\hbar} + c_2\phi_2(x)\, e^{-iE_2t/\hbar}$$

coherences

$$\psi^*\psi = |c|_1^2|\phi_1|^2 + |c|_2^2|\phi_2|^2 + c_1^* c_2 \phi_1^* \phi_2 e^{-i\omega t} + c_2^* c_1 \phi_2^* \phi_1 e^{i\omega t}$$

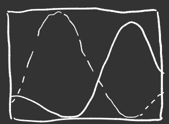
oscillation

$$W = (E_2 - E_1)/\hbar$$

electron in 1D box. $-L < x < L$, $L = \pi/2$.

$$\phi_1(x) = \sqrt{\frac{2}{\pi}}\cos x. \quad \phi_2 = \sqrt{\frac{2}{\pi}}\sin 2x.$$

$$\psi^*\psi = \frac{1}{\pi}\Big(\underbrace{1+\tfrac{1}{2}(\cos 2x - \cos 4x)}_{\text{static}} + \underbrace{(\sin 3x + \sin x)\cos\omega t}_{\text{oscillating}}\Big)$$

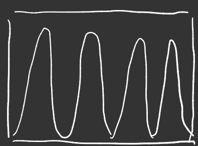


dipole moment

dipolar optical transition

... real oscillation

dipolar interaction with light...



ϕ_1 and ϕ_3. expect to see ?

dipole transitions connect opposite pairing

$\phi_1(x) = \phi_1(-x)$. parity $= 1$

$\phi_2(x) = -\phi_2(-x)$. parity $= -1$.

Energy of a dipole in magnetic field

$= \vec{d}\cdot\vec{E}$ $\quad \vec{d} = e\vec{r}$. $\quad |2\rangle\ E_{23}\ g_2$

$= e\vec{r}\cdot\vec{E}$

Consider four states $|1\rangle, |2\rangle.$ $\quad |1\rangle\ E_1\ g_1.$

$$\vec{E} = E_0 \cos\omega t$$

$$\vec{E} = \begin{pmatrix} 0 \\ 0 \\ E_0 \end{pmatrix} \cos\omega t.$$

z-axis polarized

$$\langle 2 | H_I(t) | 1 \rangle = e \langle 2 | \vec{E}\cdot\vec{r} | 1 \rangle$$

$$= e E_0 \langle 2 | \hat{z} | 1 \rangle \cos\omega t.$$

$\mu_{12} = \langle 2 | \hat{z} | 1 \rangle$ dipole moment

$n, m, l \leftarrow$ orbital angular momentum

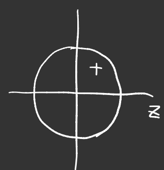
$\uparrow$ magnetic quantum number

$n = 1, 2, 3$

$n = 3.$ $\quad l = 0, 1, 2$ (s p d)

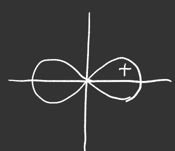
$l = 2,$ $\quad m = -2, -1, 0, 1, 2.$

$|1\rangle = |s\rangle.$



$$\mu_{12} = \langle 2 | z | 1 \rangle = \int_{R^+} dr\, \psi_2^*(r)\, z\, \psi_1(r).$$

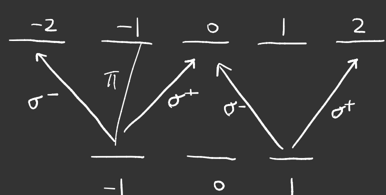
$|2\rangle = |p_z\rangle.$



$\Delta m = 0, \pm 1$

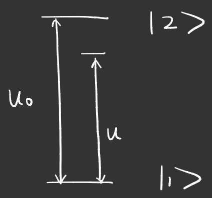
linear polarized (0) $\quad$ circular polarized (± 1)

$\Delta l = \pm 1.$



$l = 2$

$l = 1$



$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}|\psi\rangle$$

$$H = \hat{H}_0 + \hat{H}_I . \qquad \hat{H}_I = e\vec{r}\cdot\vec{E}_0 \cos\omega t .$$

$$H_0|1\rangle = E_1|1\rangle, \quad \hat{H}_0|2\rangle = E_2|2\rangle .$$

$$|\psi\rangle = U^{\dagger}|\tilde{\psi}\rangle . \qquad U = e^{-iEt/\hbar} .$$

$$H \longrightarrow UHU^{\dagger} + i\hbar UU^{\dagger} = \check{H} .$$

$$|\tilde{1}\rangle = e^{-iE_1 t/\hbar}|1\rangle, \qquad |\tilde{2}\rangle = e^{-iE_2 t/\hbar}|2\rangle .$$

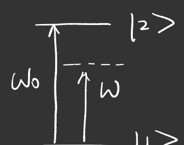
$$i\hbar \frac{d|\psi\rangle}{dt} = (H_0 + H_I)|\psi\rangle .$$

$$i\hbar\left[-\frac{iE_1}{\hbar} e^{iE_1 t/\hbar} c_1|\tilde{1}\rangle + \frac{(-iE_2)}{\hbar} e^{iE_2 t/\hbar} c_2|\tilde{2}\rangle + e^{-iE_2 t/\hbar} \dot{c}_2|\tilde{2}\rangle\right]$$

$$= E_1 e^{-iE_1 t/\hbar} c_1|\tilde{1}\rangle + E_2 e^{-iE_2 t/\hbar} c_2|\tilde{2}\rangle + H_I|\psi\rangle$$

$$= i\hbar\left[e^{iE_1 t/\hbar}\dot{c}_1|\tilde{1}\rangle + e^{-iE_2 t/\hbar}\dot{c}_2|\tilde{2}\rangle\right] = H_I\left[e^{-iE_1 t/\hbar}|\tilde{1}\rangle + e^{-iE_2 t/\hbar}|\tilde{2}\rangle\right]$$

multiply both sides by $e^{iE_1 t/\hbar}\langle\tilde{1}|$.



dipole interaction $\hat{H}_I = e\vec{r}\cdot\vec{E}\cos\omega t$.

arbitrary state $|\psi(t)\rangle = c_1(t)|\tilde{1}\rangle + c_2(t)|\tilde{2}\rangle$.

$$i\hbar \dot{c}_1 = \vec{E}_0\cdot\vec{\mu}_{12}\, e^{-i\omega_0 t}\cos(\omega t)\, c_2 . \qquad \delta\omega = \omega - \omega_0 .$$

$$= \vec{E}_0\cdot\vec{\mu}_{12}\Big(\underbrace{e^{i\delta\omega t}}_{\text{slow}} + \underbrace{e^{-i(\omega+\omega_0)t}}_{\text{rapid oscillating}}\Big)\, c_2 .$$

Under what condition. Rotating wave approx. RWA.

Integrate $\quad \frac{1}{\omega+\omega_0} \to$ small. $\quad \delta\omega$ small.

$$\dot{c}_1 = -\frac{i}{2}\frac{\vec{E}_0\cdot\vec{\mu}_{12}}{\hbar} e^{i\delta\omega t} = -\frac{i}{2}\Omega e^{i\delta\omega t} c_2 .$$

Ω : Rabi frequency.

$$\dot{c}_2 = -\frac{i}{2}\Omega e^{i\delta\omega t} .$$

fundamental process

T_1 relaxation

spontaneous emission.

(missing in semi-classical model)

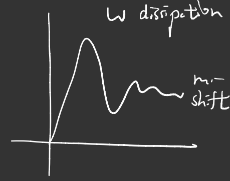
Resonant driving $\quad c_1(t=0) = 1 .$

$$\dot{c}_1 = -i\frac{\Omega}{2} c_2 .$$

$$\dot{c}_2 = -i\frac{\Omega}{2} c_1 .$$

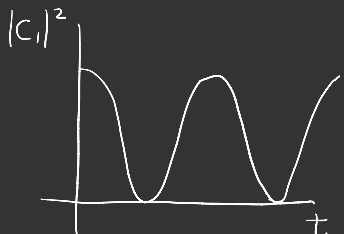
$$\ddot{c}_1 = -\frac{\Omega^2}{4} c_1 . \qquad \text{wave equation.}$$

TLS



$$c_1(t) = \cos\left(\frac{\Omega t}{2}\right) \Rightarrow |c_1(t)|^2 = \cos^2\left(\frac{\Omega t}{2}\right).$$

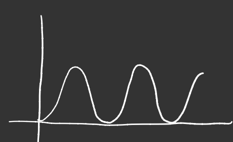
$$|c_2(t)|^2 = \sin^2\left(\frac{\Omega t}{2}\right).$$



"Rabi Oscillation"

general $\delta\omega \neq 0 .$

$$|c_2(t)|^2 = \frac{\Omega^2}{\Omega^2+\delta\omega^2}\sin^2\left(\frac{(\Omega^2+\delta\omega^2)t}{2}\right).$$

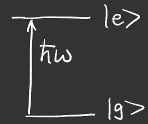


$$\hat{H}_{af} = -\hat{E}\cdot\hat{d}$$

atom-field

E field operator

electronic dipole operator



$$\hat{E}_\alpha = \hat{e}_\alpha \sum_j \left(\frac{\hbar \nu_j}{\varepsilon_0 V}\right)^2 \frac{(\hat{a}_j + a_j^+)}{\sqrt{2}} e^{ik_j\cdot c} = \sum_j \left(\hat{\mathcal{E}}_\alpha \cdot \hat{\mathcal{E}}_\alpha^+\right).$$

polarization vector

dipole

$$\hat{H}_{af} = -\hat{E}\cdot\hat{d} = -\sum_{\alpha j}\left(\hat{\mathcal{E}}_{\alpha j} + \hat{\mathcal{E}}_{\alpha j}^+\right)\left(u_\alpha |e\rangle\langle g| + u_\alpha^* |g\rangle\langle e|\right)$$

operator that change $|e\rangle \rightarrow |g\rangle$

$|g\rangle \rightarrow |e\rangle$

$\hat{a}^+|g\rangle\langle e|$ ⟹ atom decays, and emits photon

$\hat{a}|e\rangle\langle g|$ ⟹ atom excited $|g\rangle \longrightarrow |e\rangle$ and absorbing

~~$\hat{a}^+|e\rangle\langle g|$ ⟹ atom is excited $|g\rangle \longrightarrow |e\rangle$ and emit~~

~~$\hat{a}|g\rangle\langle e|$ ⟹ atom decays. and absorbs a photon~~

$$\hat{H}_{af} = -\sum_{j,\alpha}\left(\hbar g_{j\alpha}|e\rangle\langle g|\,\hat{a}_{j,\alpha} + \hbar g_{j\alpha}^*|g\rangle\langle e|\,a_{j\alpha}^+\right)$$

$$g_{j,\alpha} = \frac{-u_\alpha}{\hbar}\left(\frac{\hbar\nu_i}{2\varepsilon_0 V}\right)^{1/2} e^{i}$$ ⟸ single photon rabi frequency

$$\hat{H} = \hbar\omega|e,0\rangle\langle e,0| + \sum_{k,\alpha}\hbar\nu_k - |g_j 1_{k\alpha}\rangle\langle g_i 1_{k\alpha}| + H_{\alpha k}$$

$$|\Psi(t)\rangle = a(t)\,e^{i\omega t}|e,0\rangle.$$

$$|\psi(t)\rangle = a(t)e^{i\omega t}|e,0\rangle + \sum_{k\alpha} b_{k\alpha}(t)\,e^{-i\nu_k t}|g,1\ldots\rangle$$

perturbation

$$\dot{a} = i\sum_{k,\alpha}\hbar g_{k\alpha}\,e^{-i(\nu_k-\omega)t}\,b_{k,\alpha}$$

$$\dot{b}_{k\alpha} = -ig_{k\alpha}^*\,e^{i(\nu_k-\omega)t}\,a.$$

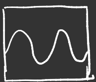
$$b_{k\alpha}(t) = ig_{k\alpha}^*\int_0^t dt'\,e^{i(\nu_k-\omega)t'}a(t') \qquad \dot{a}(t) = -\sum_{k,\alpha}|g_{k\alpha}|^2\int_0^t dt'\,e^{-i(\nu_k'-\omega)(t-t')}a(t').$$

Wigner -- approx

[illegible] limit

$$\sum_{k,\alpha} \longrightarrow \sum_{\alpha=1}^{2} \int d^2k \, D(k).$$

$$k = \left(\frac{2\pi n_1}{L}, \frac{2\pi n_2}{L}, \frac{2\pi n_3}{L} \right). \qquad \left(\frac{2\pi}{L}\right)^3 = \frac{(2\pi)^3}{V} \Rightarrow D(k) = \frac{V}{(2\pi)^3}.$$



$$\sum_{\alpha=1} \frac{V}{(2\pi)^3} \int_0^\infty k^2 dk \int_0^\pi \sin\theta \, d\theta \int_0^{2\pi} d\phi \, |g_{k\alpha}|^2 .$$

no memory of past

$$\sum_{k,\alpha} |g_{k\alpha}|^2 = \frac{\mu^2}{\square \, \varepsilon_0 \hbar c^3} \int_0^\infty \nu_k^3 \, d\nu_k$$

Monte Carol

$$\int_0^t dt' e^{-i(\nu_k-\omega)(t-t')} a(t') = \int_0^t dt' \, e^{-i\delta_k(t-t')} a(t').$$

atom continue to absorb emit ---

$$t = t'. \qquad a(t) = a(t').$$

irreversible?

$$\int_0^t dt' \, e^{-i\delta_k(t-t')} a(t') \simeq a(t) \int_0^t e^{-i\delta_k(t-t')} dt'.$$

much slower the rate field emits

$a(t)$ varies with $\gamma \ll \omega$.

$$\int_0^\infty d\tau \, e^{-i\delta_k \tau} = \pi \delta(\delta_k) - i P\left(\frac{1}{\nu_k - \omega}\right) \longleftarrow \text{lamb shift}$$

$$\dot{a}(t) = \frac{|\mu|^2 a(t)}{6\pi^2 \varepsilon_0 \hbar c^3} \int_0^\infty \nu_k^3 \, d\nu_k \, \pi \, \delta(\omega - \nu_k) = \frac{\omega^3 |\mu|^2}{6\pi^3 \varepsilon_0 \hbar c^3} a(t) = \frac{\gamma}{2} a(t).$$

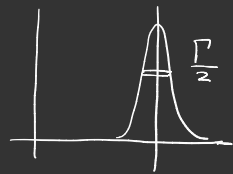
$$\gamma = \text{population decay rate} = \frac{\omega^3 |\mu|^2}{3\pi^2 \varepsilon_0 \hbar c^3}.$$

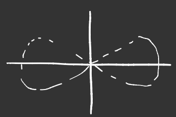
$$a'(t) = e^{-\gamma t/2}.$$

$$b_{k,\alpha}(t) = i g_{k\alpha}^* \, e^{i(\nu_k - \omega)t} \, a t = i g_k^* \, e^{i(\nu_k - \omega)t} \, e^{-\Gamma t/2}.$$

$$\lim_{t\to} b_{k\alpha}(t) = \frac{i g^*_{k\alpha}}{\Gamma/2 - i(\nu_k - \omega)}.$$

$$|b_{k\alpha}|^2 = \frac{|g_{k\alpha}|^2}{\Gamma^2/4 + (\nu_k-\omega)^2}.$$





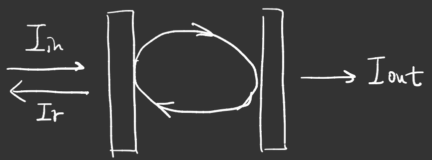
dipolar distribution of field

- - - .

Cavities, interaction with TLS

QOIC. Chapter 10.

n_0 $\quad$ n_1 $\quad$ n_0



$R_1 = |r|^2$ $\quad$ R_2

Fabry-Perot

Resonator

$$n_1 t_{01} = n_0 t_{10}.$$

$$T_{01} = \frac{n_1}{n_0} t_{01}^2 = t_{01} t_{10}.$$

$$r_{01} = -r_{10}. \quad R_{01} = R_{10} = r_{01}^2.$$

$$k = \frac{2\pi n_{index}}{\lambda}.$$

Total reflectivity

$$r = \frac{E_{reflected}}{E_{in}} = r_{01} + t_{01} r_{10} t_{10} e^{2ikd} + t_{01} r_{10} r_{10} r_{10} t_{10} e^{4ikd} \ldots$$

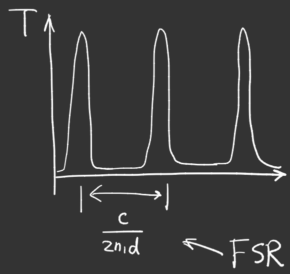
input wave interfere with wave in cavity

$$= r_{01}\left[1 + t_{01} t_{10} e^{2ikd}\left(1 + r_{10}^2 e^{2ikd} + r_{10}^4 e^{4ikd} \ldots\right)\right]$$

$$= r_{01}\left[1 - \frac{Te^{2ikd}}{1 - Re^{2ikd}}\right] = r_{01}\left[\frac{1 - e^{2ikd}}{1 - Re^{2ikd}}\right].$$

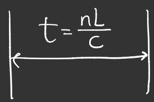
$$R = |r|^2 = \frac{4R\sin^2|kd|}{(1-R)^2 + 4R\sin^2 kd}. \qquad T = 1 - R = \frac{1}{1 + \frac{4R\sin^2 kd}{(1-R)^2}}.$$

T_{max} always equals to 1. $T_{min} = \left(\frac{1-R}{1+R}\right)^2 \approx 0$. for R approaching 1.



$T = 1$ for $f = \frac{qc}{2n_1 d}$. $q = 0, 1, 2 \dots$

finess $F = \frac{\pi (R_1 R_2)^{1/4}}{1 - (R_1 R_2)^{1/2}} = \frac{\text{peak seperation}}{\text{peak width}} = \frac{\omega_2 - \omega_1}{\Delta\omega}$



$t = \frac{nL}{c} = N \text{ -- } RN$

$t = \frac{2nL}{c} \qquad N = R^2 N.$

$\Delta N = (1-R)N$ for $t = \frac{nL_{cav}}{c}$.

decay rate $\kappa = \frac{1}{\tau_{cav}} = \Delta\omega\sqrt{R} \approx \Delta\omega$ for $R \approx 1$.

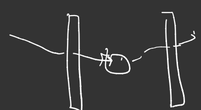
intrinsic lifetime of --- cavity

Similarly. Q = quality factor. $= \frac{\omega}{\Delta\omega}$. how long excitation lives in cavity

△ TLS interacting with cavity

γ = decay rate of atom in vacuum.

g = atom-photon coupling.



$g >> \kappa, \gamma$. strong coupling regime.

Jannis – Cummings Hamiltonian

$g << \kappa, \gamma$. weak coupling. "bad cavity". irreversable decay?

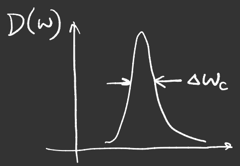
Cooperativity. C or $\eta = \frac{2g^2}{\kappa\gamma}$. $\gamma = \frac{|\mu|^2 \omega^3}{3\pi\varepsilon_0 \hbar c^3}$.

$$g = (\vec{\mu}_\alpha \cdot \vec{e})\left(\frac{\omega_k}{2\varepsilon_0 \hbar V}\right)^{1/2}.$$

improved coupling?

↳ if less than FSR.

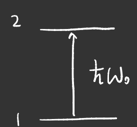
single optical mode with volume V.



$$\int_0^\infty D(\omega)\,d\omega$$

$$D(\omega) = \frac{Z}{\pi\Delta\omega_c}\frac{\Delta\omega_c}{4(\omega-\omega_c)^2+\Delta\omega^2}$$

single mode



$$D(\omega_0) = \frac{Z}{\pi\Delta\omega_c} = \frac{ZQ}{\pi\omega_0}.$$

Purcell factor

$$F_p = \frac{\gamma_{cav}}{\gamma_0}$$

$$\gamma = 2\pi\sum_{k,\alpha}|g_{k,\alpha}|^2\delta(\omega-\omega_a)$$

$$|g_{cav}|^2 = |\mu|^2\xi^2\left(\frac{\omega}{2\varepsilon_0\hbar V_{cav}}\right) \qquad \xi^2 = \frac{|\vec{\mu}\cdot\vec{e}|}{|\vec{\mu}|\cdot|\vec{e}|}.$$

$$= \frac{6\pi^2 Q^2 c^3}{\omega^2 V_{cav}}\left(\frac{\Delta\omega_c^2}{4(\omega-\omega_c)^2+\Delta\omega_c^2}\right)$$

at resonance

$$\gamma_{cav} = 2\pi\int|g_{cav}|^2\delta(\omega-\omega_c)\frac{2}{\pi\Delta\omega_c}\frac{\Delta\omega_c^2}{4(\omega-\omega_c)+\Delta\omega_c^2}.$$

$$= \frac{3(\lambda)^3}{4\pi^2}\times\frac{Q}{V}.$$

$$= 2\pi|\mu|^2\xi^2\left(\frac{\omega_a}{2\varepsilon_0\hbar V_{cav}}\right)\frac{2}{\pi\Delta\omega_c}\frac{\Delta\omega_c^2}{4(\omega_a-\omega_c)+\Delta\omega_c^2}$$

Purcell enhancement

$$C = \frac{\gamma_{cav} - \gamma}{\gamma_a}.$$

all emission into cavity

$$\beta = \frac{\gamma_{cav}}{\gamma_a + \gamma_{cav}} = \frac{F_p}{1+F_p}.$$

probability of entanglement far below 1.

deterministic interactions

reversible exchange of energy.

Single-photon / single atom interfaces : weak interactions

QE between a photon and

interfere and detect two photons with $|\psi\rangle = \frac{1}{\sqrt{2}}(|\sigma_-\rangle|+1\rangle + |\sigma_+\rangle|-1\rangle)$

probability of photon-spin entanglement generation : 10^{-6} --

Atom-cavity coupling

Dramatically modify its emission

Paper: QO with single-photon/single-atoms coupled to cavities

Optical cavity mirrors $F_p =$

Cavity tuned to

Initial demonstration of Purcell enhancement

$23S \to 22P$ transition

liminations with SC cavity? metal mirror?

heat up.

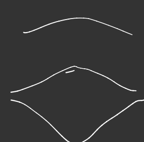
dissipation.

what happens if we shine light on?

only work well on lower frequencies.

For $T_c = 100K$, f: -- THz.

Dielectric mirrors. periodic structure.



photonic bands

npc to confine light at diffraction limit

nanoscale waveguides create guided optical

modes with -- n_{eff}. -- structure modulates n_{eff}.

modulating the --- create a defect in bandgap. (localized cavity mode)

DBR connect together

create near-diffraction limit mode volume

$$g = \sqrt{\frac{\mu_{12}^2 \omega}{2 \varepsilon_0 \hbar V_0}} = 2\pi * 1\,\mathrm{GHz}.$$

dipoles in applied electric field

Single-atom mirror

atom-photon scattering creates

π phase shift of reflected photon

Phase change in single atom

$$r_c(\eta) = \frac{(\eta-1)\gamma + 2i\delta}{(\eta+1)\gamma - 2i\delta}.$$

η: cooperativity. γ: radiative lifetime

How to detect π phase rotation. interferometer. 75%

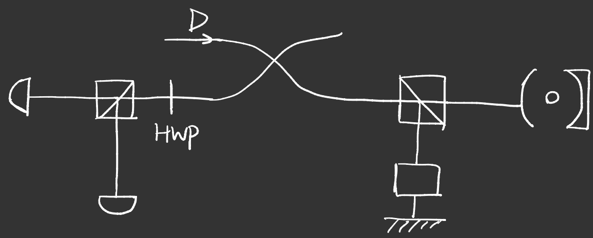
generating controllabe π phase shift with classical optics

(D.C. pockles effect)

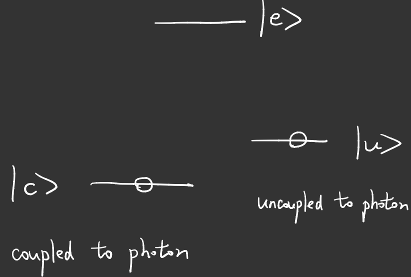
requires gigantic devices

Non-linear optics at the single photon level

Quantum non-linear optics



Quantum Phase Switch — efficiently generate spin-photon entanglement



Wed 4:30 pm @ ERC 247

Thurs 5:00 pm @ ERC 161 colloqium KPTC 106

The Strong Coupling regime

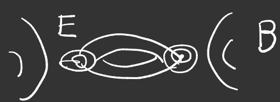
Motivation Synthesizing of arbitrary -- in SC circuits

CPWG no diffraction limit when using metal structures

No mirrors necessary

— field reflect at

end of strips



comparable to wavelength

(mode volume, strong coupling ↑)

Surface plasmon polaritons at optical frequencies

Metal. can also used to confine light at optical freq

Cooper pair box

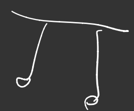
what we expect to see when bring two resonance together

avoided crossing

atom-cavity interaction cause eigenstates to repel each other

Vacuum Rabi Splitting $\omega_{\pm} = (\omega_1 - \omega_2) \pm \cdot -$

$\longrightarrow$ Classical analog on resonance $\omega \cdot\cdot =$



通过这个理解 g?

Outline

Use SC circuit and qubit to climb the Jaynes-Cummings Ladder

Generation of Fock States using SC quantum circuit

qubit can be tuned in and out of resonance rapidly

Coherent oscillation

dynamically evolving system

wait half Rabi freq and then detune

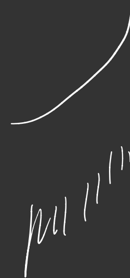
What if I bring Qubit in excited state into resonance with cavity in a single photon Fock state

Climbing the JC ladder

repeatedly and controllably inject single photons into resonator

coherent state coupling - - -

of all frequencies



Rabi freq

prob density

$$P_n(a) = \frac{a^n e^{-a}}{n!}$$

Liminations in atomic clocks

quantum projection noise $\sim \sqrt{N}$

$\uparrow \quad \downarrow$

Spin Squeezing

actually use measurements to generate entanglement atom 1 $\quad \uparrow - \downarrow$

atom 2 $\quad \uparrow + \downarrow$

Total spin

Measure.

$J = 1 \quad \uparrow\uparrow$

$J = -1 \quad \downarrow\downarrow$

$J = 0 \quad \uparrow\downarrow$ or $\downarrow\uparrow$

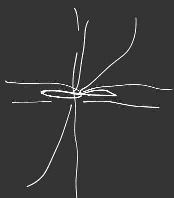
shift $\longrightarrow$

info about J

Final Presentation

1. Squeezed light from a sillicon micromechanical resonator

↳ on chip measurements for microresonators



homodyne noise spectrum

squeezed and unsqueezed

2. Broadband waveguide memory for entangled photons

photon-photon entanglement ⇌ photon collective -- excitation

generation of photon-photon entanglement

PPLN crystal

Fiber Brag grating and etalon to further filter out rogue wavelengths

storing in memory

low temp... give rise to several energy states

Retrieval of ... photon

AFC

state tomography

find --- state ?

measure of entanglement CHSH

in different basis

3. Robust multi-qubit --- node -- error correction

Selective readout of electron spin qubit

Measure reflected intensity

electron spin-dependent phase

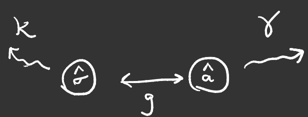
PHONE gate with error detection

of ...

nuclear spin-photon entanglement

laser induced nuclear coherence

4. Resolving energy levels of a nano-mechanical oscillator



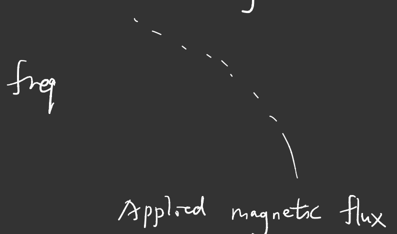
the quantum acoustic approach

no exchange energy

QND detection

phononic crystal lattice & bandgap

qubit spectroscopy and mechanical modes



5. Single-photon Distillation via a photonic parity measurement

parity regime Coherent pulse

Wigner function

6. Detecting spins by their fluorescence with a MW photon counter

∘ phase-coherent spin echos detected by quadrature measurement

∘ incoherent photons difficult to detect

7. Principle of spin detection with a photon counter

SMPD. application in quantum sensing

8. Stablization and characterization of a kerr cat qubit

use QND readout to measere coherence

Fast quantum control = process tomography

9. Quantum-dot spin-photon entanglement via frequency down-conversion

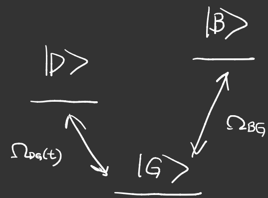
conditional probability

10. Deterministic creation of entangled atom-light -- cat

11. Experimental demonstration of memory enhanced quantum communication

QBER Bell inequality violation

12. Quantum jump



$$\chi = \frac{g^2}{\Delta}.$$

dispersive coupling of cavity ...

reversing the quantum jump

phonon-mediated quantum state transfer and ...

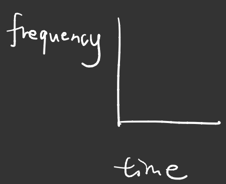
optimized pulses

reverse trajectory

13. Photon mediated quantum state transfer and remote qubit entanglement

deterministic emission and capture of phonons

Q.



Is information really being exchanged?

$|e_0\rangle \longrightarrow |g_1\rangle + |e_0\rangle$

quantum state swap!

coupling of phonons to -- photons

14. Single molecule strong coupling at room temperature

NPoM nano cavities.

proof of single molecule strong coupling poisson distribution

15. Quantum electromagnetism · — topological waveguide

affects the global behavior of · — on topological band

observation of chiral symmetry protection

Quantum state transfer via edge states

16. entanglement of 2 quantum memories via km - - -

atom-photon entanglement

17. Observation of coherent optical information storage in an atomic medium using halted light pulses

→ significant geometry change under charge state transition

N deep donor in diamond   large amount of phonon coupling

B-N pairs.   large   HR (electron-phonon coupling)

large broadband emission

hard to form

substitial   interstitial

PBE. HSE.

DAP exhibit large dipole moments   we can leverage

maximum localized wannier functions

spectral density function

ES HR factor

↓

smaller electron-phonon coupling

phonon coupling for B-N pairs in 3C-SiC

Al-N → shallower defect

→ deeper acceptor

smaller amount of geometrical distortion

which phonon contribute?   broader EPL?

4H & 6H-SiC

Group Meeting Kai

Strong coupling in 2D materials

Giant effective Zeeman splitting

optical response with monolayer TMDCs

in magnetic field

Control of spin and valley psedo spin

trion configuration

emission/absorption

$MoSe_2$ WSe_2

shift in PL: stark shift

Stoke?

linear pump

depolarize electrons

optical pump to control the spin distribution

optical nonlocal control

Group Meeting

metabolomics ?

Tung High field NMR using diamond quantum sensors

14.1 T

sensor target properties control

E.g. Sensor ωS_z

Target $\gamma S_z B(t)$

Control $\Omega(t) \cos\omega t$

△ Ramsey DC magnetic field

ODMR T^* limit

$\rightarrow T$ limit

effect of high field on sample

△ Hahn echo AC B field

△ NMR

Compare NV & Coil

Spectral Resolution

NV ~ mHz coil ~ 10 mHz

why NV ?

Sensitivity

NV ~ $pT/\sqrt{Hz}$ Coil $2fT/\sqrt{Hz}$

Sample Volume ☑

Single ~~molecule~~ protein/cell biology

Why high field?

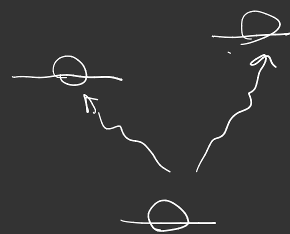
larger chemical shift

higher thermal polarization

longer T_2

$H_0 = \Delta S_z^2 + \gamma_e B_0 S_z$.

spin Manipulation



Challenges

Coherent spin manipulation

111 Diamond Membrane

High field sensing scheme

Lab on a chip (LoC)

lossy transmission

resistive loss

absorption by water

Power vs Modulation?

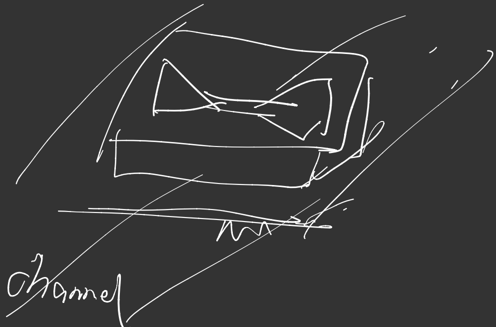
Amplifier - Multipler Chain

Quasi-optics + corrugated waveguide

HE11 waveguide

HE11 taper

enhance local B field



flow channel

+ lock in

PD → correlation → Qdyne

Harmonic Mixer

$f_{IF} = f_{RF} \bmod f_{LO}$.

BW : $1/N\tau$. limited by T_2.

ultra high resolution

replace interrogation time with quantum memory

higher order → noisy

111 diamond

AERIS scheme

$\eta \propto \frac{1}{C\sqrt{N\tau}}$.

↳ also have higher geometrical factor.

Fabrication

Polish removal rate is low

solution: syntek

MPCVD twins stacking fault

step-flow growth

dislocation impurity

microfluidic chips