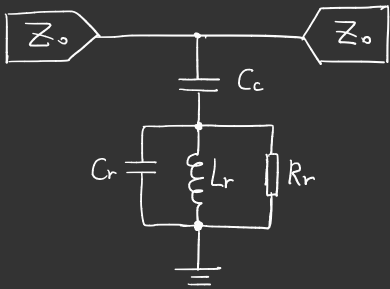


$$S_{11} = \frac{Z_r - Z_0}{Z_r - Z_0}.$$

$$Z_r = \frac{1}{i\omega C_c} + \left(\frac{1}{R_r} + \frac{1}{i\omega L_r} + i\omega C_r\right)^{-1}$$

$$\Rightarrow \quad S_{11} = \frac{1 - \omega^2 L_r\left[C_r + C_c\left(1 - \frac{Z_0}{R_r}\right)\right] + i\omega\left[\frac{L_r}{R_r} - Z_0 C_c\left(1 - \omega^2 L_r C_r\right)\right]}{1 - \omega^2 L_r\left[C_r + C_c\left(1 + \frac{Z_0}{R_r}\right)\right] + i\omega\left[\frac{L_r}{R_r} + Z_0 C_c\left(1 - \omega^2 L_r C_r\right)\right]}$$

At resonance impedance vanishes:

$$\frac{1}{i\omega_0 C_c} + \frac{1}{i\omega_0 C_r + \frac{1}{i\omega_0 L_r}} = 0.$$

resonance freq $\Rightarrow$ $\omega_0 = 1/\sqrt{L_r(C_r + C_c)}$.

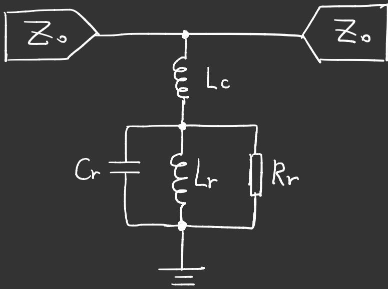
Assuming $R_r \gg Z_0$, on resonance:

$$|S_{11}(\mathrm{res})| = \left|\frac{\frac{L_r}{Z_0 R_r C_r} - \frac{x^2}{1+x}}{\frac{L_r}{Z_0 R_r C_r} + \frac{x^2}{1+x}}\right|,$$

with $x = \frac{C_c}{C_r}$. ratio of $\frac{\text{coupling } C}{\text{resonator } C}$

Z: characteristic impedance of resonator

$$= \left|\frac{\frac{Z^2}{Z_0}\frac{1}{R_r} - \frac{x^2}{1+x}}{\frac{Z^2}{Z_0}\frac{1}{R_r} + \frac{x^2}{1+x}}\right|.$$



$$S_{11} = \frac{Z_r - Z_0}{Z_r - Z_0}.$$

$$C_c \rightarrow -\frac{1}{\omega^2 L_c}$$

$$Z_r = i\omega L_c + \left(\frac{1}{R_r} + \frac{1}{i\omega L_r} + i\omega C_r\right)^{-1}$$

$$\Rightarrow \quad S_{11} = \frac{1 - \omega^2 L_r C_r + \frac{L_r}{L_c}\left(1 - \frac{Z_0}{R_r}\right) + i\left[\omega\frac{L_r}{R_r} + \frac{Z_0}{\omega L_c}\left(1-\omega^2 L_r C_r\right)\right]}{1 - \omega^2 L_r C_r + \frac{L_r}{L_c}\left(1 + \frac{Z_0}{R_r}\right) + i\left[\omega\frac{L_r}{R_r} - \frac{Z_0}{\omega L_c}\left(1-\omega^2 L_r C_r\right)\right]}$$

At resonance impedance vanishes :

$$i\omega_0 L_c + \frac{1}{i\omega_0 C_r + \frac{1}{i\omega_0 L_r}} = 0.$$

resonance freq $\Rightarrow$ $\omega_0 = \sqrt{\frac{1}{C_r}\left(\frac{1}{L_c} + \frac{1}{L_r}\right)}$

Assuming $R_r \gg Z_0$, on resonance :

$$|S_{11}(\mathrm{res})| = \left|\frac{\frac{L_r}{Z_0 R_r C_r} - \frac{1}{x(x+1)}}{\frac{L_r}{Z_0 R_r C_r} + \frac{1}{x(x+1)}}\right|,$$

ratio of $\frac{\text{coupling } L_c}{\text{resonator } L_r}$

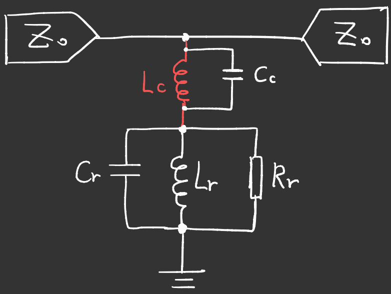
with $x = \frac{L_c}{L_r}$.

Z : characteristic impedance of resonator

$$= \left|\frac{\frac{Z^2}{Z_0}\frac{1}{R_r} - \frac{1}{x(x+1)}}{\frac{Z^2}{Z_0}\frac{1}{R_r} + \frac{1}{x(x+1)}}\right|.$$

length $\uparrow$ $L_c \uparrow$

diameter $\downarrow$ $L_c \uparrow$



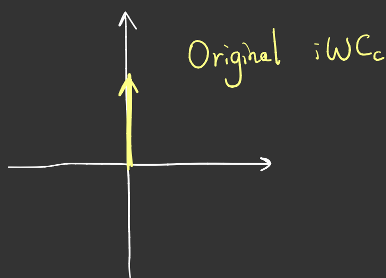
$$S_{11} = \frac{Z_r - Z_o}{Z_r - Z_o}$$

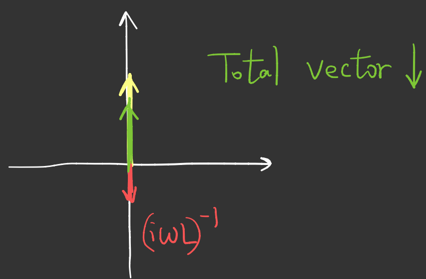
length ↑ L_c ↑

diameter ↓ L_c ↑

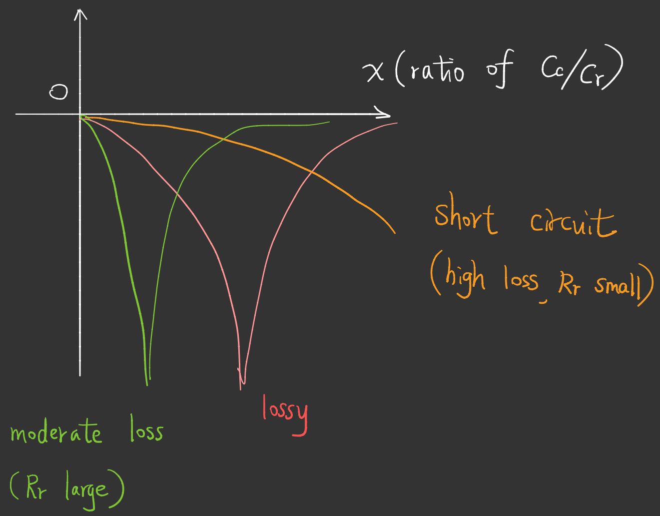
$$Z_r = \left[\left(\frac{1}{i\omega C_c}\right)^{-1} + (i\omega L)^{-1}\right]^{-1} + \left(\frac{1}{R_r} + \frac{1}{i\omega L_r} + i\omega C_r\right)^{-1}$$

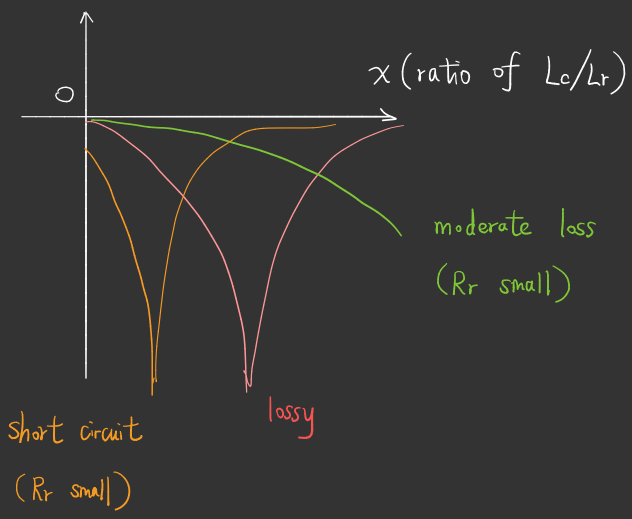
Is actually substuting $\frac{1}{i\omega C_c} \longrightarrow \left[\left(\frac{1}{i\omega C_c}\right)^{-1} + (i\omega L)^{-1}\right]^{-1}$





Is actually increasing C_c ! $C_c' = C_c + \frac{1}{\omega^2 L}$


0
x (ratio of C_c/C_r)
short circuit
(high loss, R_r small)
lossy
moderate loss
(R_r large)


0
x (ratio of L_c/L_r)
moderate loss
(R_r small)
lossy
short circuit
(R_r small)