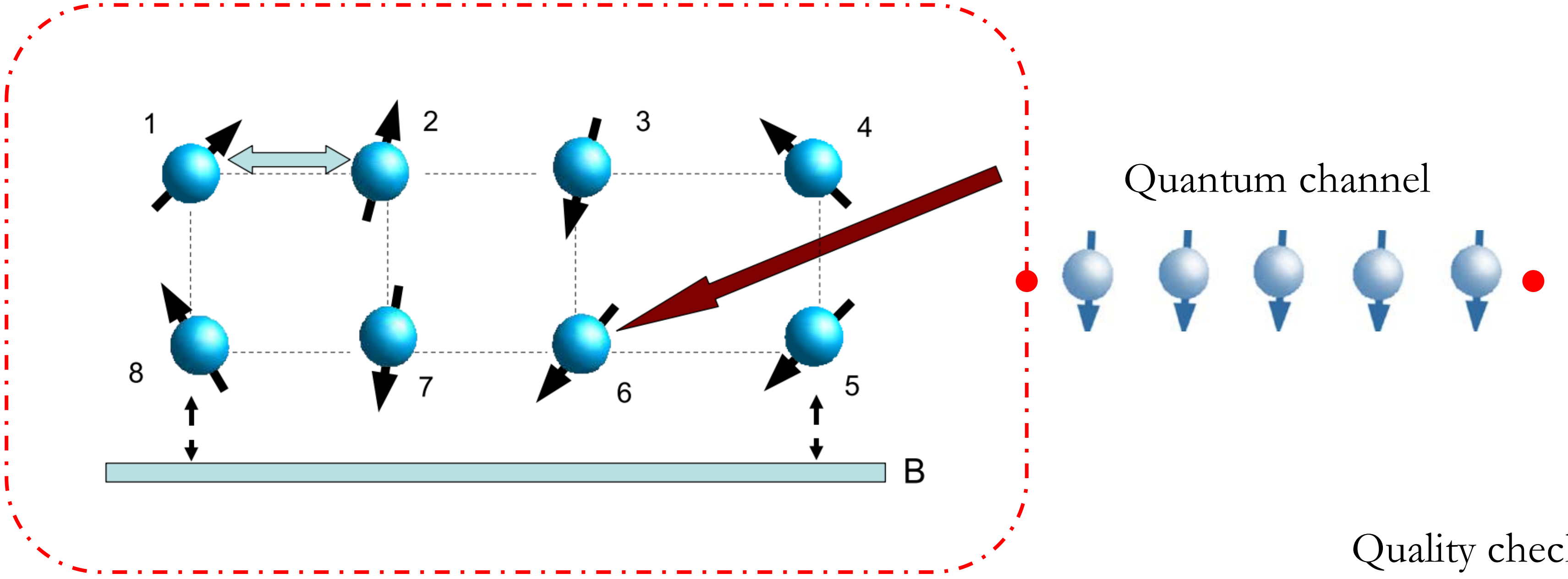
Quantum State Transfer

Through Spin Chain Dynamics

Motivation

Limitation



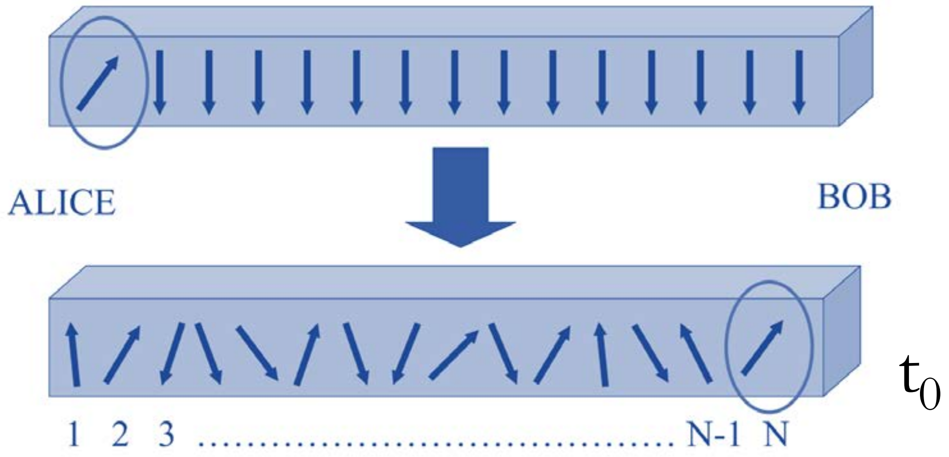
Quality check for a qubit array

Quantum response to a quantum impulse

Outline

- Initialized spin chain
- Engineered spin chain
- Random spin chain
- Discussion & related works

Initialized spin chain as quantum channel



isotropic Heisenberg Hamiltonian H_{XYZ}

$$\mathbf{H_G} = -\sum_{<i,j>} J_{ij}\ \vec{\sigma}^i . \vec{\sigma}^j - \sum_{i=1}^{N} B_i \sigma_z^i .$$

$J_{ij} < 0$ FM

$J_{ij} > 0$ AFM

$$|\mathbf{0}\rangle = |000...0\rangle$$

$$|\mathbf{j}\rangle = |00...010....0\rangle$$

$$\mathbf{j = 1, 2, ..s, ..r, .., N}$$

$$F = \langle\psi_{in}|\rho_{out}(t_0)|\psi_{in}\rangle$$

Highest fidelity using LOCC ? $\frac{2}{3}$

$F\downarrow$ with number of spins N

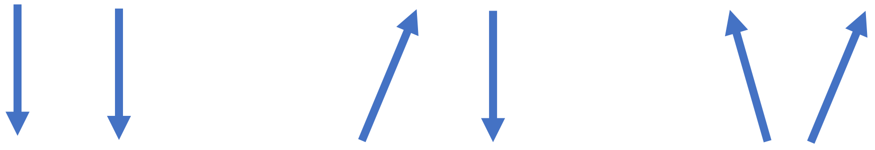
Comparable with classical when N~80

$$F = \frac{|f_{r,s}(t_0)|\cos\gamma}{3} + \frac{|f_{r,s}(t_0)|^2}{6} + \frac{1}{2}$$

$$\gamma = \arg\{f_{r,s}(t_0)\}$$

$$f_{r,s}(t) = \langle\mathbf{r}|\exp\{-i\mathbf{H_G}t\}|\mathbf{s}\rangle$$

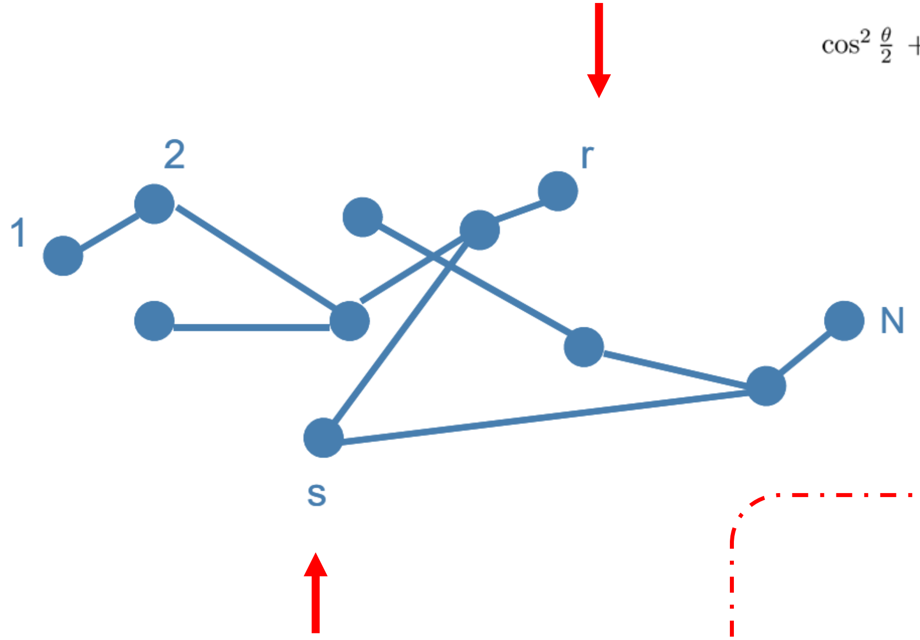
Two spin case? H=J(XX+YY+ZZ)



Arbitrary graph of spins

$$|\psi_{out}(t)\rangle = \frac{1}{\sqrt{P(t)}}(\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}f^N_{s,r}(t)|1\rangle)$$

$\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}|f^N_{r,s}(t)|^2$



$$|\psi_{in}\rangle = \cos\frac{\theta}{2}\ |0\rangle + e^{i\phi}\sin\frac{\theta}{2}\ |1\rangle$$

Amplitude damping channel

$$M_0 = \begin{bmatrix} 1 & 0 \\ 0 & |f^N_{r,s}(t_0)| \end{bmatrix} \qquad M_1 = \begin{bmatrix} 0 & \sqrt{1-|f^N_{r,s}(t_0))|^2} \\ 0 & 0 \end{bmatrix}$$

$\mathcal{E} = |f^N_{r,s}(t_0)|$ concurrence

1D chain: $E_m = 2B + 2J(1-\cos\{\frac{\pi}{N}(m-1)\}$

$$f_{r,s}(t) = \sum_{m=1}^{N} a_m \cos\{\frac{\pi}{2N}(m-1)(2r-1)\}e^{-iE_m t_0}\cos\{\frac{\pi}{2N}(m-1)(2s-1)\}$$

Remarks:

1. Entanglement can be distributed to arbitrary distances
2. Opposite site of 2N spin ring ~ N spin line
3. Time t_0 is not simply ~ N/J

S. Bose, Phys. Rev. Lett 91, 207901(2003); V. Subrahmanyam, Phys. Rev. A 69, 034304 (2004)

Wave-packet encoding

"Propagation" of quantum state:

$$\Psi(x,t) = \begin{cases} \delta(x) & t=0 \\ \sqrt{\frac{m}{2\pi\hbar|t|}}e^{-i\mathrm{sgn}(t)\pi/4}\exp\left[i\frac{mx^2}{2\hbar t}\right] & t\neq 0 \end{cases}$$

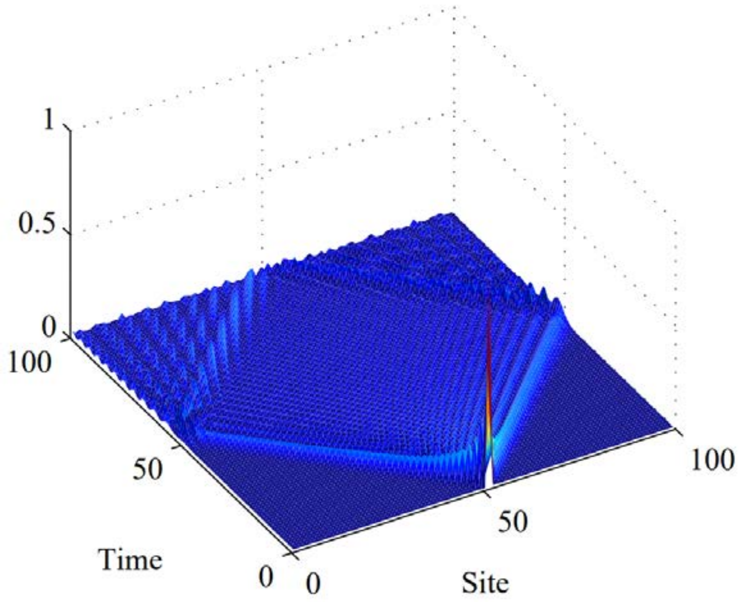


FIG. 1: Propagation of the state $|N/2\rangle$ in a 100-site Heisenberg spin ring.

"truncated" gaussian wavepacket states:

$$|G(j_0,k)\rangle = \sum_j e^{-(j-j_0)^2/L^2} e^{-ik_0 j}|\mathbf{j}\rangle$$

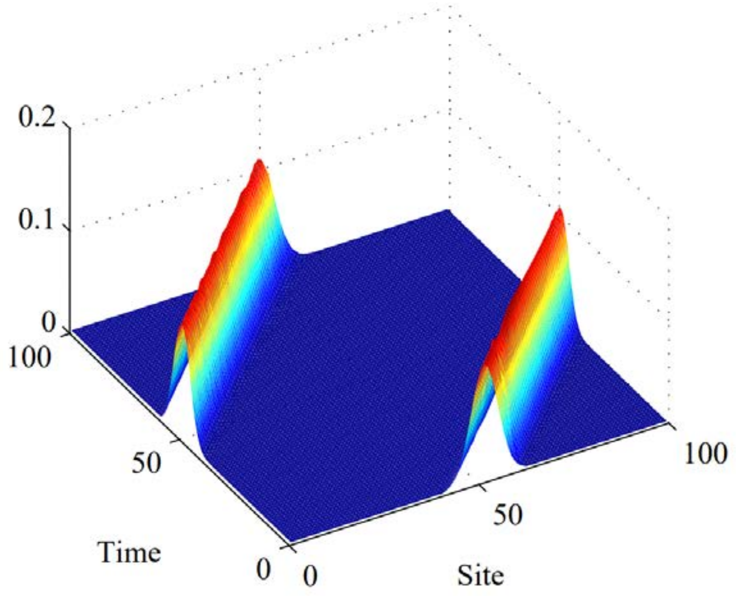


FIG. 2: Propagation of a truncated gaussian-modulated W-state of width 10 and wavenumber $N/4$ in a 100-site Heisenberg spin ring.

https://physics.stackexchange.com/questions/129978/the-dirac-delta-function-as-an-initial-state-for-the-quantum-free-particle

T. J. Osborne and N. Linden, Phys. Rev. A 69, 052315 (2004).

Engineered spin chain

mirror inverted wave function

$$\psi(x,t) = \sum_k c_k e^{-iE_k t}\phi_k(x)$$

$$E_k \propto k^2 \qquad \phi_k(-x) = (-1)^k \phi_k(x)$$

commensurate E　　alternating parity

$$\psi(x,\tau) = \sum_k c_k (-1)^k \phi_k(x) = \sum_k c_k \phi_k(-x) = \psi(-x)$$

$$E_k \tau = k^2 \pi$$

Perfect state transfer in mirrored chain

$$\psi(x) = \sum_j c_j |\mathbf{j}\rangle$$

$$J_{j,j+1} = \sqrt{j(N-j)} \qquad J_{j,j+1} = J_{N-j,N-j+1}$$

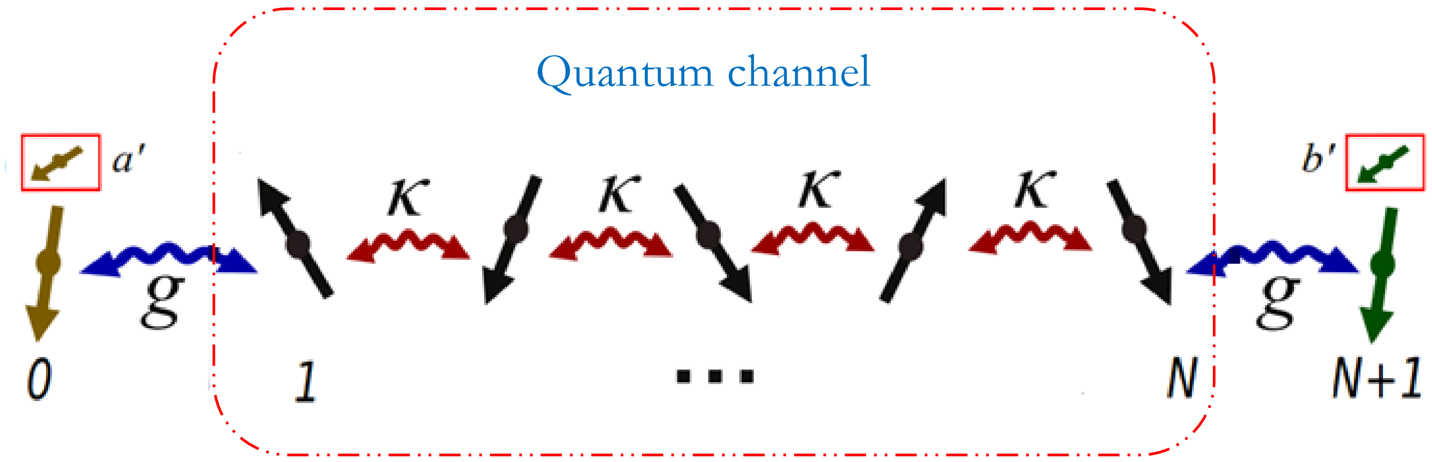
$$E_k \propto k$$

commensurate E　　alternating parity

$$\psi(x,\tau) = \sum_j c_j |\mathbf{N}-\mathbf{j}\rangle$$

M. Christandl, N. Datta, A. Ekert and A. J. Landahl, Phys. Rev. Lett. 92, 187902 (2004).

Random spin chain



$g \ll \kappa$

$$H = H_0 + H'$$

$$\sum_{i=1}^{N-1} \kappa(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+) \qquad g(S_0^+ S_1^- + S_{N+1}^+ S_N^- + \text{ h.c.})$$

specific collective eigenmode

long-range coherent interaction

$\longrightarrow$ Robust state transfer

Fermionization

	Interaction	Excitation	Collective phenomenon
boson	Spin-Spin	magnon	Spin wave
fermion	Electron-Electon	quasielectron	-

$$c_i = e^{i\pi \sum_0^{i-1} S_j^+ S_j^-} S_i^-$$

Jordan-Wigner transformation

$$f_k^\dagger = \frac{1}{A} \sum_{j=1}^{N} \sin \frac{jk\pi}{N+1} c_j^\dagger$$

$$\rightarrow H_0 = \sum_{i=1}^{N-1} \kappa (c_i^\dagger c_{i+1} + c_i c_{i+1}^\dagger)$$

$$H_0 = \sum_{k=1}^{N} E_k f_k^\dagger f_k \qquad E_k = 2\kappa \cos \frac{k\pi}{N+1}$$

$$H' = \sum_{k=1}^{N} t_k (c_0^\dagger f_k + (-1)^{k-1} c_{N+1}^\dagger f_k + \text{ h.c.})$$

$$t_k = \frac{g}{A} \sin \frac{k\pi}{N+1}$$ tunneling rate

Free fermion state transfer (FFST)

$$H_0 = \sum_{k=1}^{N} E_k f_k^\dagger f_k \qquad E_k = 2\kappa \cos \frac{k\pi}{N+1}$$

zero energy fermionic mode

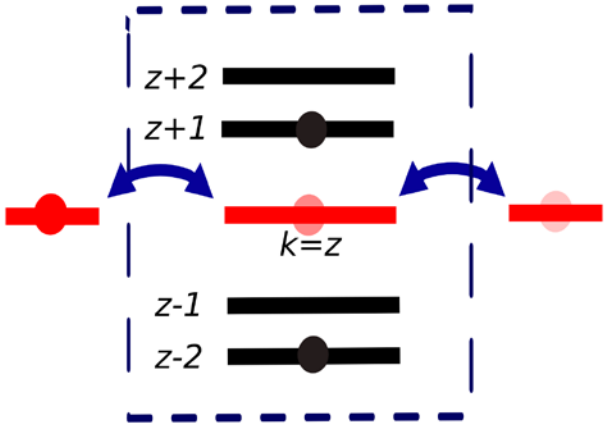
$$k = z \equiv (N+1)/2$$

$$H' = \sum_{k=1}^{N} t_k (c_0^\dagger f_k + (-1)^{k-1} c_{N+1}^\dagger f_k + \text{ h.c.})$$

Effective Hamiltonion $\tau = \frac{\pi}{\sqrt{2} t_z}$

$$t_k = \frac{g}{A} \sin \frac{k\pi}{N+1}$$ tunneling rate

$$H_{eff} = t_z (c_0^\dagger f_z + c_{N+1}^\dagger f_z + \text{ h.c.})$$



$$U_{eff} = e^{-i\tau H_{eff}} = (-1)^{f_z^\dagger f_z}(1 - (c_0^\dagger + c_{N+1}^\dagger)(c_0 + c_{N+1}))$$

$$U_{eff}^{fermi} = (-1)^{n_0 + n_{N+1} + n_z}(-1)^{n_0 n_{N+1}} \text{SWAP}_{0,N+1}$$

$$\{(1, c_0^\dagger, c_{N+1}^\dagger, c_0^\dagger c_{N+1}^\dagger)|00\rangle_{0,N+1}\}$$

Remarks: 1.End fermions entangled through CPHASE

2.Infidelity from off-resonant modes

Related works

Many-body localization protected quantum state transfer

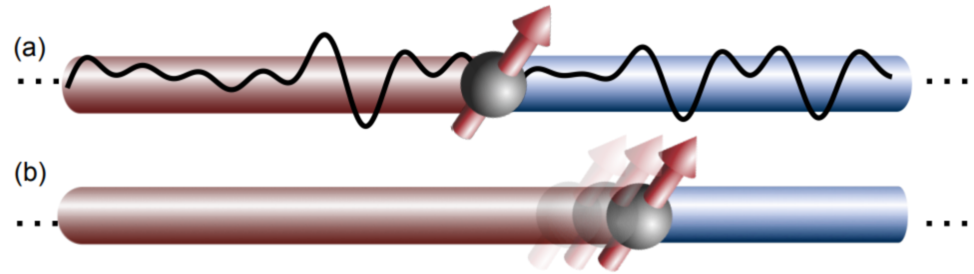
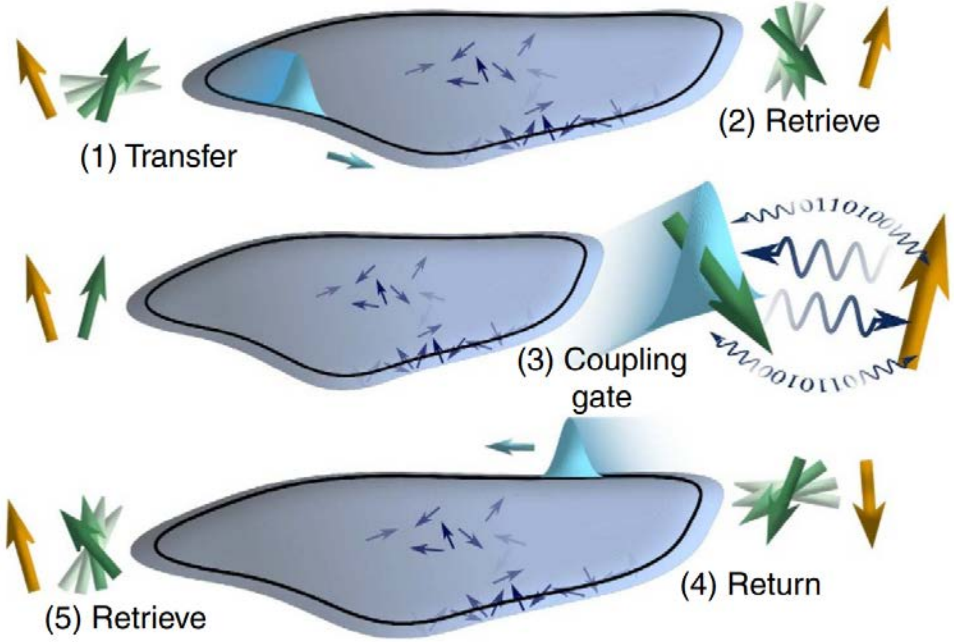


FIG. 1. (a) Schematic representation of the boundary between a trivial and SPT insulator. A single qubit is bound at the interface. Strong disorder (black line) leads to many-body localization in both phases. (b) Shuttling of the edge qubit occurs via shifts in the trivial-SPT boundary.

Yao, Norman Y., Chris R. Laumann, and Ashvin Vishwanath. "Many-body localization protected quantum state transfer." arXiv preprint arXiv:1508.06995 (2015).

Topologically protected quantum state transfer



Yao, Norman Y., et al. "Topologically protected quantum state transfer in a chiral spin liquid." Nature communications 4.1 (2013): 1585.