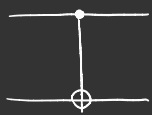
No Cloning Theorem

例 $|\psi\rangle = a|0\rangle + b|1\rangle$

$|0\rangle$



$a|00\rangle + b|11\rangle$

$$(a|0\rangle + b|1\rangle)|0\rangle = a|00\rangle + b|11\rangle.$$

compare.

$$|\psi\rangle|\psi\rangle = a^2|00\rangle + ab|01\rangle + ba|10\rangle + b^2|11\rangle.$$

证明 $\boxed{|\psi\rangle}$ 数据槽 $\boxed{|s\rangle}$ 目标槽

克隆装置的初态

$$|\psi\rangle \otimes |s\rangle.$$

复制过程由 U 决定

$$|\psi\rangle \otimes |s\rangle \xrightarrow{U} U(|\psi\rangle \otimes |s\rangle) = |\psi\rangle \otimes |\psi\rangle.$$

假设复制过程适用于两个特定纯态 $|\psi\rangle$. $|\varphi\rangle$. 有

$$U(|\psi\rangle \otimes |s\rangle) = |\psi\rangle \otimes |\psi\rangle.$$

$$U(|\varphi\rangle \otimes |s\rangle) = |\varphi\rangle \otimes |\varphi\rangle.$$

取内积得

$$\langle\psi|\varphi\rangle = (\langle\psi|\varphi\rangle)^2. \quad \Rightarrow \quad |\psi\rangle = |\varphi\rangle \text{ 或 } |\psi\rangle. |\varphi\rangle \text{ 正交.}$$

非正交量子态不可克隆.

试图克隆混合态？

克隆装置不是酉？

允许偏差会怎样？

与可达信息的联系

量子态的可达信息 $< H(p)$ 的结果

非正交态的可达信息总小于制备的熵

Schmidt 分解

可以适当地选择 H^A、H^B 的基向量使 $\mathcal{H}$ 中的两体纯态表示为

$$|\psi\rangle = \sum_{i=0}^{\min\{d_A,d_B\}-1} c_i |e_i\rangle \otimes |f_i\rangle. \qquad c_i > 0.$$

任意的 $2\otimes2$ 量子纯态

$$|\psi\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle. \qquad \sum_{i\mu} |c_{i\mu}|^2 = 1.$$

$\rho = |\psi\rangle\langle\psi|$. $\rho^A = \mathrm{Tr}_B(\rho)$. 设 ρ^A 的两个正交归一本征态为 $|e_0\rangle$、$|e_1\rangle$. 相应本征态为 p_0. p_1. 且 p_0. $p_1 > 0$. $p_0 + p_1 = 1$.

对粒子A作幺变换

$$|0\rangle \longrightarrow |e_0\rangle \ , \ |1\rangle \longrightarrow |e_1\rangle$$

$$|\psi\rangle = |e_0\rangle|\tilde{\varphi}_0\rangle + |e_1\rangle|\tilde{\varphi}_1\rangle.$$

现希望证明 $|\tilde{\varphi}_0\rangle$. $|\tilde{\varphi}_1\rangle$ 彼此正交.

将 ρ 表示为

$$\rho = |e_0\rangle\langle e_0| \otimes |\tilde{\varphi}_0\rangle\langle\tilde{\varphi}_0| + |e_0\rangle\langle e_1| \otimes |\tilde{\varphi}_0\rangle\langle\tilde{\varphi}_1| + |e_1\rangle\langle e_0| \otimes |\tilde{\varphi}_1\rangle\langle\tilde{\varphi}_0| + |e_1\rangle\langle e_1| \otimes |\tilde{\varphi}_1\rangle\langle\tilde{\varphi}_1|$$

$$\mathrm{Tr}\left[|\tilde{\varphi}_i\rangle\langle\tilde{\varphi}_j|\right] = \langle 0|\tilde{\varphi}_i\rangle\langle\tilde{\varphi}_j|0\rangle + \langle 1|\tilde{\varphi}_i\rangle\langle\tilde{\varphi}_j|1\rangle = \langle\tilde{\varphi}_j|\tilde{\varphi}_i\rangle. \qquad i,j = 0,1.$$

$$\rho^A = |e_0\rangle\langle e_0|\langle\tilde{\varphi}_0|\tilde{\varphi}_0\rangle + |e_0\rangle\langle e_1|\langle\tilde{\varphi}_1|\tilde{\varphi}_0\rangle + |e_1\rangle\langle e_0|\langle\tilde{\varphi}_0|\tilde{\varphi}_1\rangle + |e_1\rangle\langle e_1|\langle\tilde{\varphi}_1|\tilde{\varphi}_1\rangle$$

在表象 $\{|e_0\rangle, |e_1\rangle\}$ 中 ρ^A 是对角的.

$$\rho^A = p_0 |e_0\rangle\langle e_0| + p_1 |e_1\rangle\langle e_1|.$$

将 ρ^A 上述两种形式对比可得

$$\langle\tilde{\varphi}_i|\tilde{\varphi}_j\rangle = \delta_{ij} p_i.$$

将 $|\tilde{\varphi}_i\rangle$ 归一化. 令 $|f_i\rangle = \frac{1}{\sqrt{p_i}}|\tilde{\varphi}_i\rangle$.

最终 $|\psi\rangle = \sqrt{p_0}|e_0\rangle|f_0\rangle + \sqrt{p_1}|e_1\rangle|f_1\rangle.$

混合态的纯化

考虑量子系统 Q 处于混合态 ρ^Q. 引入辅助系统 A.

需构造纯态 $|\Psi\rangle \in \mathcal{H}$. 使

$$\rho^Q = \mathrm{Tr}_A(|\Psi\rangle\langle\Psi|)$$

H^Q 上的 ρ^Q 到 $\mathcal{H}$ 中的 $|\Psi\rangle$. 纯化.

标准纯化形式

$$\rho^Q = \sum_i \lambda_i |\xi_i\rangle\langle\xi_i|$$

H^Q 维数为 n. ρ^Q 满秩. λ_i 为 ρ^Q 本征值. $\sum_i \lambda_i = 1$.

H^A 维数选定为 n. $\dim H^Q = \dim H^A = n$. H^A 基向量选为 $\{|\varphi_i\rangle\}$. 构造

$$|\Psi\rangle = \sum_i \sqrt{\lambda_i}\, |\xi_i\rangle \otimes |\varphi_i\rangle.$$

易验证 $\mathrm{Tr}_A(|\Psi\rangle\langle\Psi|) = \rho^Q$.

非标准纯化

ρ^Q 为 $\{p_i, \psi_i\}_{i=1,\cdots,m}$ 的平均量子态. 系统 Q 的密度矩阵为

$$\rho^Q = \sum_i p_i \psi_i. \qquad \psi_i = |\psi_i\rangle\langle\psi_i|.$$

引入辅助系统 A. $\dim(H^A) = m$. $|\varphi_i\rangle \in \mathcal{H}^A$ 且 $\langle\varphi_i|\varphi_j\rangle = \delta_{ij}$.

可将 ρ^Q 纯化形式表示为

$$|\Psi\rangle = \sum_i \sqrt{p_i}\, |\psi_i\rangle \otimes |\varphi_i\rangle.$$

在 H^A 的另一组基向量上表示 $|\Psi\rangle$.

对辅助系统的局部酉变换

$$\{|\varphi_i\rangle\} \xrightarrow{V} \{|\eta_i\rangle\}.$$

V 酉矩阵. $v_{ij} = \langle\varphi_i|\eta_j\rangle$.

新表象中

$$|\Psi\rangle = \sum_i \sqrt{\lambda_i}\, |\xi_i\rangle \otimes \sum_j |\eta_j\rangle\langle\eta_j|\varphi_i\rangle = \sum_j \Big(\sum_i \sqrt{\lambda_i}\, v_{ij}^* |\xi_i\rangle\Big) \otimes |\eta_j\rangle.$$

任意正算子都自动是厄米的。有对角表示

正算子 $\to$ $\forall |v\rangle$ 有 $\langle v|A|v\rangle \geq 0$.

谱分解定理

(如果一个算子是正规的，当且仅当它可对角化)

向量空间 V 上的任意正规算子 M 在 V 的某组正交基下都是可对角化的.

反之任意可对角化的算子都是正规的.

d 维空间 V. $d=1$. ✓

设 λ 为 M 的一个特征值. P 是到 λ 本征空间上的投影. Q 是到其正交补上的投影.

$$M = (P+Q)M(P+Q) = PMP + QMQ + QMP + PMQ.$$

显然 $PMP = \lambda P$.　　M 把子空间 P 映到自身. $QMP = 0$.

$PMQ = 0$ 也成立. 设 $|v\rangle$ 为子空间 P 中的一个元素.

$$MM^\dagger|v\rangle = M^\dagger M|v\rangle = \lambda M^\dagger|v\rangle$$

有特征值 λ. 是子空间 P 中的一个元素

故 $QM^\dagger P = 0$. 取伴随 $PMQ = 0$.

故 $M = PMP + QMQ$. 证明它正规

注意到 $QM = QM(P+Q) = QMQ$.

且 $QM^\dagger = QM^\dagger(P+Q) = QM^\dagger Q$.

故根据 M 的正规性并注意到 $Q^2 = Q$. 有

$$\begin{aligned} QMQQM^\dagger Q &= QMQM^\dagger Q \\ &= QMM^\dagger Q = QM^\dagger MQ = QM^\dagger QMQ \\ &= QM^\dagger QQMQ. \end{aligned}$$

故 QMQ 正规.

QMQ 在 Q 的某个标准正交基下是可对角化的. PMP 在 P ... 也 ...

因此 $M = PMP + QMQ$ 相对于全向量空间下某标准正交基可对角化.

极式分解和奇异值分解

Polar　　　　Singular

将一般的线性算子分解为西算子和正算子的乘积

极式分解

设 A 是向量空间 V 上的一个线性算子，那么存在西算子 U 和正算子 J, K 使得

$$A = UJ = KU.$$

左极式　右极式

其中 $J=\sqrt{A^\dagger A}$，$K=\sqrt{AA^\dagger}$．且 J, K 唯一．若 A 可逆，U 唯一．

$J=\sqrt{A^\dagger A}$ 是一个正算子．谱分解 $J=\sum_i \lambda_i |i\rangle\langle i|$ $(\lambda_i \geq 0)$

定义 $|\psi_i\rangle = A|i\rangle$．　$\langle\psi_i|\psi_i\rangle = \lambda_i^2$．　若 $i\neq j$，$\langle e_i|e_j\rangle = \langle i|A^\dagger A|j\rangle/\lambda_i\lambda_j = \langle i|J^2|j\rangle/\lambda_i\lambda_j = 0$．

只考虑 $\lambda_i\neq 0$ 的 i．定义 $|e_i\rangle = |\psi_i\rangle/\lambda_i$．故 $|e_i\rangle$ 归一化，且正交．

扩展 $|e_i\rangle$ 使之成为一组标准正交基，也用 $|e_i\rangle$ 表示．

定义 $U=\sum_i |e_i\rangle\langle i|$．　$\lambda_i\neq 0$ 时有　$UJ|i\rangle = \lambda_i|e_i\rangle = |\psi_i\rangle = A|i\rangle$．

$\lambda_i = 0$ 时有　$UJ|i\rangle = 0 = |\psi_i\rangle$．

A 与 UJ 在基 $|i\rangle$ 上作用一样．$A=UJ$．

J 唯一．因 $A=UJ$ 乘以 $A^\dagger = JU^\dagger$ 得 $J^2 = A^\dagger A$．　$J=\sqrt{A^\dagger A}$，且唯一．

若 A 可逆，则 J 可逆．U 由 $U=AJ^{-1}$ 唯一确定．

$A = UJ = UJU^\dagger U = KU$．其中 $K=UJU^\dagger$ 为正算子．

又因 $AA^\dagger = KU^\dagger UK = K^2$．有 $K=\sqrt{AA^\dagger}$．

奇异值分解

设 A 为方阵. 存在酉矩阵 $U.V$. 非负对角阵 D. 使

$$A = UDV.$$

D 的对角元素称 A 的奇异值.

根据极式分解. 对酉矩阵 S 和半正定矩阵 J 有 $A=SJ$.

根据谱分解定理. 对酉矩阵 T 和非负对角阵 D 有 $J=TDT^\dagger$.

令 $U \equiv ST$ 和 $V = T^\dagger$ 即证.

量子噪声与量子操作

量子操作形式体系

经典噪声与 Markov 过程

$$\begin{bmatrix} q_0 \\ q_1 \end{bmatrix} = \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \end{bmatrix}$$

$$\vec{q} = E\vec{p}.$$

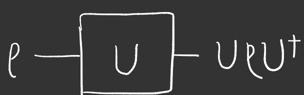
量子操作 $\rho' = \mathcal{E}(\rho)$. 例：酉变换. 测量

$$\mathcal{E}(\rho) = U\rho U^\dagger. \qquad \mathcal{E}_m(\rho) = M_m \rho M_m^\dagger.$$

三种理解. 1. 系统与环境相互作用

2. 算子和表示

3. 带有物理动机的公理



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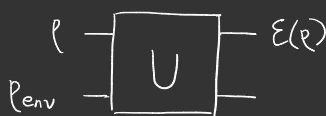


开放

量子态的变换 $\mathcal{E}(\rho) = \mathrm{Tr}_{env}\left[U(\rho\otimes\rho_{env})U^\dagger\right].$

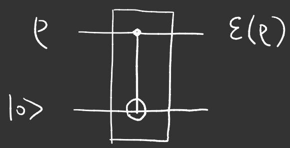
不一定是直积态

如 U 不含与环境相互作用

$$\mathcal{E}(\rho) = \hat{U}\rho\hat{U}^\dagger.$$

若主系统是有 d 维 Hilbert 空间. 那么环境在不超过 d^2 维的 Hilbert 空间中建模即可. ?

若环境有 ∞ 自由度. 如何确定 U?



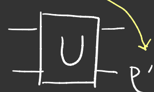
$$\mathcal{E}(\rho) = P_0 \rho P_0 + P_1 \rho P_1 .$$

$U = CNOT.$ $\quad Tr_{env}\left[U(\rho \otimes |0\rangle\langle 0|)U^{\dagger}\right]$

更普适.

qubit $\quad A \xrightarrow{制备}$ 未知态 ρ.

qutrit $\quad B \longrightarrow$ 标准态 $|0\rangle$.



$$\Big\} \quad U(\rho \otimes |0\rangle\langle 0|)U^{\dagger}$$

描述此过程的量子操作为 $\mathcal{E}(\rho) = \rho' = Tr_A\left(U(\rho \otimes |0\rangle\langle 0|)U^{\dagger}\right)$.

算子和表示

$$\mathcal{E}(\rho) = \sum_k \langle e_k | U \left[\rho \otimes |e_0\rangle\langle e_0|\right] U^{\dagger} | e_k\rangle$$

$$= \sum_k E_k \rho E_k^{\dagger}. \qquad E_k = \langle e_k | U | e_0\rangle.$$

完备性关系 $\quad 1 = Tr(\mathcal{E}(\rho)) = Tr\left(\sum_k E_k \rho E_k^{\dagger}\right) = Tr\left(\sum_k E_k^{\dagger} E_k \rho\right).$

对所有 ρ 成立. 有 $\sum_k E_k^{\dagger} E_k = I.$ (保迹量子操作).

非保迹: $\sum_k E_k^{\dagger} E_k \leq 1.$

物理诠释 $\quad$ 在基矢 e_k 上执行对环境的测量 (只影响环境的状态, 而不改变主系统的状态)

$$\rho_k \propto Tr_E\left(|e_k\rangle\langle e_k| U(\rho \otimes |e_0\rangle\langle e_0|)U^{\dagger} |e_k\rangle\langle e_k|\right)$$

结果k

$$= \langle e_k | U(\rho \otimes |e_0\rangle\langle e_0|) U^{\dagger} | e_k\rangle = E_k \rho E_k^{\dagger}.$$

归一化 $\quad \rho_k = \dfrac{E_k \rho E_k^{\dagger}}{Tr(E_k \rho E_k^{\dagger})}.$

k 出现的概率

$$p(k) = Tr\left(|e_k\rangle\langle e_k| U(\rho \otimes |e_0\rangle\langle e_0|) U^\dagger |e_k\rangle\langle e_k|\right) = Tr\left(E_k \rho E_k^\dagger\right).$$

有 $\mathcal{E}(\rho) = \sum_k p_k \rho_k = \sum_k E_k \rho E_k^\dagger$. 用于操作元 $\{E_k\}$ 的量子操作

选取态 ρ. 以概率 $Tr(E_k \rho E_k^\dagger)$ 用 ρ_k 随机地替换它.

两体纯态纠缠变换

Alice、Bob 共享一对纠缠态 $|\psi\rangle$，并可在各自系统上执行任意操作和测量。

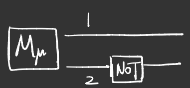
但相互间只能经典通信。 $|\psi\rangle \longrightarrow |\varphi\rangle$?

例. Bell state $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$

Alice 执行 M_1、M_2. $M_1 = \begin{bmatrix} \cos\theta & \\ & \sin\theta \end{bmatrix}$. $M_2 = \begin{bmatrix} \sin\theta & \\ & \cos\theta \end{bmatrix}$.

测量后状态 $|\psi_\mu\rangle = \frac{M_\mu|\psi\rangle}{\sqrt{\langle\psi|M_\mu^\dagger M_\mu|\psi\rangle}}$. $|\psi_1\rangle = \frac{\cos\theta|00\rangle+\sin\theta|11\rangle}{\sqrt{\cos^2\theta+\sin^2\theta}} = \cos\theta|00\rangle+\sin\theta|11\rangle$.

$|\psi_2\rangle = \sin\theta|00\rangle + \cos\theta|11\rangle$.



$\cos\theta|00\rangle+\sin\theta|11\rangle$.

优超

对 d 维实向量进行排序以期得到一个向量或多或少地区别于另一个向量。

$x = (x_1, x_2, \cdots x_d)$ $x^\downarrow$ 降序. $x_1^\downarrow$ 最大.

$y = (y_1, y_2, \cdots y_d)$

若对 $\forall\, k=1,\cdots d-1$ 都有 $\sum_{j=1}^{k} x_j^\downarrow \leq \sum_{j=1}^{k} y_j^\downarrow$ 成立当且仅当 $k=d$ 时取等号.

称 x 优超于 y. $x \prec y$. $\prec$ $\prec$ $\prec$

设 $|\psi\rangle$、$|\varphi\rangle$ 是 Alice-Bob 联合系统上的状态. 定义

$\rho_\varphi = Tr_B(|\psi\rangle\langle\psi|)$. $\rho_\psi = Tr_B(|\varphi\rangle\langle\varphi|)$.

λ_ψ、λ_φ 特征值构成的向量.

$|\psi\rangle$ 可通过 LOCC 转化为 $|\varphi\rangle$ 的充要条件是 $\lambda_\psi \prec \lambda_\varphi$.

一些事实

$x \prec y$ 的充要条件是 x 可以写成 y 的置换的一个凸组合.

当 x 比 y 更无序, x 可由置换 y 的元素并混合结果向量得到. 那么 $x \prec y$.

命题 $x \prec y$ 当且仅当 $x = \sum_j p_j P_j y$.

设 $x \prec y$. 令 $x = x^{\downarrow}$, $y = y^{\downarrow}$.

$x_1, x_2, \cdots x_{d+1}$

$y_1 \cdots y_{j-1}, y_j, \cdots y_{d+1}$

归纳假设 $x = \sum_j p_j P_j y$. $d=1$ ✓.

设 x, y 为 $d+1$ 维向量且 $x \prec y$. 那么 $x_1 \le y_1$.

选择 j 使 $y_j \le x_1 \le y_{j-1}$. 定义 $t \in [0,1]$ 并使

$$x_1 = t y_1 + (1-t) y_j.$$

定义置换的凸组合为 $D = tI + (1-t)T$. 于是

$$Dy = (x_1, y_2, \cdots y_{j-1}, (1-t)y_1 + t y_j, y_{j+1}, \cdots y_{d+1})$$

定义 $x' \equiv (x_2, \cdots x_{d+1})$, $y' \equiv (y_2, \cdots y_{j-1}, (1-t)y_1 + t y_j, y_{j+1}, \cdots y_{d+1})$

可证明 $x' \prec y'$.

…

△ 假设$|\psi\rangle$可由LOCC转化为$|\varphi\rangle$. 那么这一转化可由以下两步协议实现

Alice 执行一个由M_j描述的本地测量. 并把测量结果j发给 Bob.

Bob再在自己系统上执行一个酉操作U_j.

证明Bob所做的任意测量都可由Alice模拟.

假设Bob执行M_j作用在纯态$|\psi\rangle$上. $|\psi\rangle = \sum_l \sqrt{\lambda_l}|l_A\rangle|l_B\rangle$.

若 $M_j = \sum_{k,l} M_{j,kl} |k_B\rangle\langle l_B|$. 定义 $N_j \equiv \sum_{k,l} M_{j,kl}|k_A\rangle\langle l_A|$.

Bob 测量后状态 $|\psi_j\rangle \propto M_j|\psi\rangle = \sum_{kl} M_{j,kl}\sqrt{\lambda_l}\,|l_A\rangle|k_B\rangle$. 概率 $\sum_{kl}\lambda_l|M_{j,kl}|^2$.

Alice $|\varphi_j\rangle \propto N_j|\psi\rangle = \sum_{kl} M_{j,kl}\sqrt{\lambda_l}\,|k_A\rangle|l_B\rangle$. $\sum_{kl}\lambda_l|M_{j,kl}|^2$.

同一状态.

有相同 Schmidt 系数. 存在U_j. V_j使

$|\psi_j\rangle = (U_j\otimes V_j)|\varphi_j\rangle$. 因此Bob的$M_j$. 等价于$V_j$后$U_jN_j$.

定理 两体纯态$|\psi\rangle$能通过LOCC转化到另一纯态$|\varphi\rangle$当且仅当$\lambda_\psi \in \lambda_\varphi$.

假设转化过程为Alice先执行M_j. 测量结果发给Bob. 再执行U_j.

对 Alice. $M_j\rho_\psi M_j^\dagger = p_j\rho_\varphi$.

极式分解$M_j\sqrt{\rho_\psi}$指存在酉矩阵V_j使

$$M_j\sqrt{\rho_\psi} = \sqrt{M_j\rho_\psi M_j^\dagger}\,V_j = \sqrt{p_j\rho_\varphi}\,V_j.$$

$$\Rightarrow \sum\sqrt{\rho_\psi}M_j^\dagger M_j\sqrt{\rho_\psi} = \sum p_j V_j^\dagger\rho_\varphi V_j.$$

$$\Rightarrow \rho_\psi = \sum_j p_j V_j^\dagger\rho_\psi V_j.$$

$\lambda_\psi \prec \lambda_\varphi$.

量子 Fourier 变换

离散 Fourier 变换：

x_0, x_1

$y_0 = \frac{1}{\sqrt{2}} x_0 + \frac{1}{\sqrt{2}} x_1$

$y_1 = \frac{1}{\sqrt{2}} x_0 - \frac{1}{\sqrt{2}} x_1$

x_0, x_1, x_2

$y_0 = \frac{1}{\sqrt{3}} x_0 + \frac{1}{\sqrt{3}} x_1 + \frac{1}{\sqrt{3}} x_2$

$y_1 = \frac{1}{\sqrt{3}} x_0 + \frac{1}{\sqrt{3}} x_1 e^{\frac{2\pi i}{3}} + \frac{1}{\sqrt{3}} x_2 e^{4\pi i/3}$

$y_2 = \frac{1}{\sqrt{3}} x_0 + \frac{1}{\sqrt{3}} x_1 e^{\frac{4\pi i}{3}} + \frac{1}{\sqrt{3}} x_2 e^{8\pi i/3}$.

$y_4 = \frac{1}{\sqrt{3}} x_0 + \frac{1}{\sqrt{3}} x_1 e^{8\pi i/3} + \frac{1}{\sqrt{3}} x_2 e^{16\pi i/3}$ × 也可以？

Input：复向量 $x_0, x_1, \cdots x_{N-1}$

Output：$y_0, y_1, \cdots y_{N-1}$

$$y_k \equiv \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} x_j e^{2\pi i jk/N}.$$

Quantum Fourier Transform：

定义在 $|0\rangle, \cdots |N-1\rangle$ 上的算子

$$|j\rangle \longrightarrow \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{2\pi i jk/N} |k\rangle.$$

Unitary. (Proof?)

$$\sum_{j=0}^{N-1} x_j |j\rangle \longrightarrow \sum_{k=0}^{N-1} \underbrace{\sum_{j=0}^{N-1} \frac{1}{\sqrt{N}} e^{2\pi i jk/N} x_j}_{y_k} |k\rangle = \sum_{k=0}^{N-1} y_k |k\rangle.$$

取 $N = 2^n$. $|0\rangle, \cdots |2^n - 1\rangle$.

$|j\rangle$ 写为 $j = j_1 j_2 \cdots j_n$. $j = j_1 2^{n-1} + j_2 \cdot 2^{n-2} + \cdots j_n 2^0$.

$$|j_1 \cdots j_n\rangle \longrightarrow \frac{1}{\sqrt{2^n}} \sum_{k=0}^{2^n-1} e^{2\pi i jk/2^n} |k\rangle$$ 写为二进制

$|0\rangle + e^{2\pi i j/4} |1\rangle$

$+ e^{4\pi i j/4} |2\rangle$

$|3\rangle \to |11\rangle$ $+ e^{6\pi i j/8} |3\rangle$

$e^{6\pi i j/8} |1\rangle |1\rangle$

$e^{(2^1+2^0) \cdot 2\pi i j/2^2} |1\rangle |1\rangle$

$$= \frac{1}{\sqrt{2^n}} \sum_{k=0}^{2^n-1} e^{2\pi i j (k_1 2^{n-1} + k_2 2^{n-2} + \cdots k_n 2^0)/2^n} |k_1 k_2 \cdots k_n\rangle$$

什么样的过程呢？整体来看.

$$= \frac{1}{2^{\frac{n}{2}}} \sum_{k_1=0}^{1} \sum_{k_2=0}^{1} \cdots \sum_{k_n=0}^{1} e^{2\pi i j \left(\sum_{l=1}^{n} k_l 2^{-l} \right)} |k_1 \cdots k_n\rangle$$

$$= \frac{1}{2^{n/2}} \sum_{k_1=0}^{1} \cdots \sum_{k_n=0}^{1} \bigotimes_{l=1}^{n} e^{2\pi i j k_l 2^{-l}} |k_l\rangle$$

$$= \frac{1}{2^{n/2}} \bigotimes_{l=1}^{n} \left[\sum_{k_l=0}^{1} e^{2\pi i j k_l 2^{-l}} |k_l\rangle \right] = \frac{1}{2^{n/2}} \bigotimes_{l=1}^{n} \left[|0\rangle + e^{2\pi i j 2^{-l}} |1\rangle \right].$$

$$|0\rangle + e^{2\pi i j/4}|1\rangle$$

$$+ e^{4\pi i j/4}|2\rangle$$

$$|3\rangle \rightarrow |11\rangle \qquad + e^{6\pi i j/8}|3\rangle$$

$$e^{6\pi i j/8}|1\rangle|1\rangle$$

$$e^{(2^1+2^0)\cdot 2\pi i j/2^2}|1\rangle|1\rangle$$

$$\frac{1}{2}\left(|0\rangle + e^{2\pi i j/4}|1\rangle + e^{4\pi i j/4}|2\rangle + e^{6\pi i j/8}|3\rangle\right)$$

$$= \frac{1}{2}\left(e^{(0\cdot 2^1+0\cdot 2^0)\cdot 2\pi i j/4}|00\rangle + e^{(0\cdot 2^1+1\cdot 2^0)\cdot 2\pi i j/4}|01\rangle + e^{(1\cdot 2^1+0\cdot 2^0)^{2\pi i j/4}}|10\rangle\right.$$

$$\left. + e^{(1\cdot 2^1+1\cdot 2^0)^{2\pi i j/4}}|11\rangle\right)$$

$$= \frac{1}{2}\sum_{k_1=0}^{1}\sum_{k_2=0}^{1} e^{2\pi i j\left(\sum_{l=1}^{2} k_l 2^{-l}\right)}|k_1 k_2\rangle$$

$$e^{2\pi i j(k_1\cdot 2^{-1}+k_2\cdot 2^{-2})}|k_1 k_2\rangle$$

$$= \sum_{k_1}\sum_{k_2} e^{2\pi i j\, k_1\cdot 2^{-1}}|k_1\rangle\, e^{2\pi i j\, k_2\cdot 2^{-2}}|k_2\rangle.$$

$$= \bigotimes_{l=1}^{2}\left[e^{2\pi i j\, 0\cdot 2^{-l}}|0\rangle + e^{2\pi i j\cdot 1\cdot 2^{-l}}|1\rangle\right]$$

$\langle x | \psi_{x_0} \rangle$ wavepacket around x_0

translation of particle : $U_\ell = \exp(-iP\ell/\hbar)$.

$\nearrow$ $S_z |\uparrow\rangle = \frac{\hbar}{2} |\uparrow\rangle$. $S_z |\downarrow\rangle = -\frac{\hbar}{2} |\downarrow\rangle$.

$$|\Psi\rangle = \alpha^{\uparrow} |\uparrow\rangle \otimes |\psi^{\uparrow}\rangle + \alpha^{\downarrow} |\downarrow\rangle \otimes |\psi^{\downarrow}\rangle.$$

$$U = \exp(-2i\, S_z \otimes P\ell).$$

$|\uparrow\rangle \otimes |\psi^{\uparrow}_{x_0}\rangle \Rightarrow |\uparrow\rangle \otimes |\psi^{\uparrow}_{x_0-\ell}\rangle$

$$(\alpha^{\uparrow}|\uparrow\rangle + \alpha^{\downarrow}|\downarrow\rangle) \otimes |\psi^{\uparrow}_{x_0}\rangle \Rightarrow \alpha^{\uparrow}|\uparrow\rangle \otimes |\psi_{x_0-\ell}\rangle + \alpha^{\downarrow}|\downarrow\rangle \otimes |\psi_{x_0+\ell}\rangle$$

measure in rotational basis $\{|S_+\rangle, |S_-\rangle\}$?

$$S = \sum \left(|x+1\rangle\langle x| \otimes |\uparrow\rangle\langle\uparrow| + |x-1\rangle\langle x| \otimes |\downarrow\rangle\langle\downarrow| \right)$$

$$S(I \otimes H)|x\rangle|c\rangle$$

$$= S\left(|x\rangle \otimes H|c\rangle\right) = \frac{1}{\sqrt{2}} S\left[|x\rangle \otimes \left(|\uparrow\rangle \pm |\downarrow\rangle\right)\right]$$

$$= \frac{1}{\sqrt{2}} |x+1\rangle|\uparrow\rangle \pm \frac{1}{\sqrt{2}}|x-1\rangle|\downarrow\rangle$$

$$S\left(|x\rangle \otimes |\uparrow\rangle\right) = |x+1\rangle|\uparrow\rangle. \qquad S\left(|x\rangle \otimes |\downarrow\rangle\right) = |x-1\rangle|\downarrow\rangle.$$

$$H|\uparrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$$

$$H|\downarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle - |\downarrow\rangle\right).$$

$$U^3|0\rangle|\uparrow\rangle = ?$$

$$S\left(|0\rangle \otimes H|\uparrow\rangle\right) = S\left(|0\rangle \otimes \left(\frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}\right)\right) = \frac{1}{\sqrt{2}}\left(|1\rangle|\uparrow\rangle + |-1\rangle|\downarrow\rangle\right)$$

initial state $|i\rangle$

Probability of detecting an event corresponding to final state $|f\rangle$

$p = |\langle f|i\rangle|^2$. If initial state modified by an intermediate

unitary interaction $\hat{U}(\varepsilon)$. $P_\varepsilon = |\langle f|\hat{U}(\varepsilon)|i\rangle|^2$.

$U(\varepsilon)$ 生成元为 $\hat{A}$. $\hat{A}$ the (infinitesimal) generator of $U(\varepsilon)$

A: impulsive H of product form ?

if ε small enough. $U(\varepsilon)$ is weak.

$$P_\varepsilon = |\langle f|U(\varepsilon)|i\rangle|^2 = |\langle f|1 - i\varepsilon\hat{A} + \cdots|i\rangle|^2$$

$$\langle f|1 - i\varepsilon A|i\rangle\langle i|1 - i\varepsilon A|f\rangle$$

$$\langle f|i\rangle\langle i|1 - i\varepsilon A|f\rangle - \langle f|i\varepsilon A|i\rangle\langle i|1 - i\varepsilon A|f\rangle$$

$$= P_\varepsilon - \langle f|i\rangle\langle i|i\varepsilon A|f\rangle - \underline{\langle f|i\varepsilon A|i\rangle}\langle i|f\rangle$$

weak interaction regime $\qquad -i\varepsilon\langle f|A|i\rangle\langle i|f\rangle$.

$$= P_\varepsilon + \underline{2\varepsilon\, \mathrm{Im}\langle i|f\rangle\langle f|A|i\rangle} + O(\varepsilon^2)$$

Full Taylor expansion for P_ε/P is completely characterized by

complex weak values A_w^n of all orders n.

laser $|i\rangle|\psi_i\rangle$

initial polarization state   state of transverse beam profile

$$P = |\langle f|i\rangle|^2 |\langle \psi_f|\psi_i\rangle|^2.$$

final transverse state postselected by each pixel

unperturbed

$\varepsilon = \tau\nu$. each polarization displaced by an equal amount

$$\hat{U}(\tau) = e^{-i\tau\hat{H}/\hbar}. \qquad \hat{H} = \nu\hat{S}\otimes\hat{p}.$$

Gaussian beam ...

初态. 经HWP. QWP

$$|i\rangle = \frac{|H\rangle - e^{i\phi}|V\rangle}{\sqrt{2}}$$

pass through a postselection polarizer

oriented at a small angle from diagonal state

偏振态 $\phi = 0.1$?

$$|f\rangle = \cos\frac{\theta}{2}|H\rangle + \sin\frac{\theta}{2}|V\rangle.$$

$$\theta = \frac{\pi}{2} - 0.2$$

nearly orthogonal.

$P\downarrow$. 后放大.

Heisenburg scaling

先介绍定义

$$\sum M_m^\dagger M_m = I$$

Projective Measurement

$$p(m) = \langle \psi | \hat{M}_m^\dagger \hat{M}_m | \psi \rangle.$$

$$|\psi_m\rangle = \hat{M}_m |\psi\rangle.$$

If $M_m^\dagger M_m = M_m$,

is projective.

Generalize. POVM.

Weak Measurement

ancilla measurement

uncertainty of

outcome be larger than differences between eigenvalues of system

S

$$A|a_j\rangle = a_j|a_j\rangle$$

$$g(t)\hat{A} \otimes \hat{P}_d$$

$$|\psi\rangle \otimes |\phi(x)\rangle$$

$$e^{-iHt/\hbar} |\psi\rangle \otimes |\phi(x)\rangle$$

$|\phi_d\rangle$ wave function of measurement device

In position basis $|\phi\rangle = |\phi_d\rangle = \int_x \phi(x)\,|x\rangle\,dx.$

$$\phi(x) = (2\pi\sigma^2)^{-\frac{1}{4}} e^{-x^2/4\sigma^2}.$$

$$[S_i^z, S_j^{\pm}] = 2\delta_{ij} S_i^{\pm} \qquad [a_i, a_j^+] = \delta_{ij}$$

$$[S_i^+, S_j^-] = 2\delta_{ij} S_i^z. \qquad [a_i, a_j] = [a_i^+, a_j^+] = 0$$

多种理解的方法

① excitation. spin → Boson → fermonic

② 1D non-interacting Hubbard model

③ SDW in spin chain

Interaction	Excitation	Wave
Spin-Spin	magnon?	spin wave
Electron-Electron	准电子	—

QECC (quantum error correcting code)

$$|\bar{0}\rangle = \left[\frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)\right]^{\otimes 3}$$

$$|\bar{1}\rangle = \left[\frac{1}{\sqrt{2}}\left(|000\rangle - |111\rangle\right)\right]^{\otimes 3}$$

logical 0 and 1

can be distinguished by

$$\sigma_x^{(1)} \otimes \sigma_x^{(2)} \otimes \sigma_x^{(3)}$$

$X_1 X_2 X_3$ operator

nonlocally encoded!

provides protection against noise

Suppose an unknown quantum state prepared and encoded as

$$a|\bar{0}\rangle + b|\bar{1}\rangle$$

If a single bit flip occurs.

The location of bit flip can be determined by measuring

the two-qubit operators

(prevent damage)

$Z_1 Z_2$, $Z_2 Z_3$. should be collective measurement

Could recover the error by flipping that qubit back.

ancilla qubit in $|0\rangle$

Initial state $a|0\rangle + b|1\rangle$. $\xrightarrow{\text{encoded}}$ $a|000\rangle + b|111\rangle$.

a simple two stage error-correction procedure

(1) Error detection or syndrome diagnosis

A measurement tells what error occurred.

Measurement result called error syndrome.

The syndrome measurement does not cause change to the state.

because we only know the error but do not know a or b.

(2) Recovery

Use the value of error syndrome to know the procedure to recover the initial state.

Improving the error analysis

Fidelity. between a pure and mixed state

$$F(|\psi\rangle, \rho) = \sqrt{\langle\psi|\rho|\psi\rangle}$$

$|\psi\rangle$. 没有纠错码时，通过 channel 后状态为

$$\rho = (1-p)|\psi\rangle\langle\psi| + pX|\psi\rangle\langle\psi|X.$$

$$F = \sqrt{(1-p) + p\underbrace{(\langle\psi|X|\psi\rangle)^2}_{\geq 0}}$$

So, $F \geq \sqrt{1-p}$.

比较了有无纠错时的量子态，证明 F 提高

假设被保护的状态为 $|\psi\rangle = a|0_L\rangle + b|1_L\rangle$.

noise 和 error-correction 以后的状态为 ？如何写出

$$\rho = \underbrace{\left[(1-p)^3 + 3p(1-p)^2\right]}|\psi\rangle\langle\psi| + \cdots$$

纠错过程成功完成的概率.（3个 qubit 中 ≤1 个发生翻转）

解释了：省略项表示 2 或 3 个 bit flip 的贡献，均为正.

$$F \geq \sqrt{(1-p)^3 + 3p(1-p)^2}.$$

如果不是用四个投影算符，用 Z_1Z_2 或 Z_2Z_3

可理解为比较 1.2，2.3 是否相同.

Z_1Z_2 有谱分解

$$Z_1Z_2 = (|00\rangle\langle 00| + |11\rangle\langle 11|)\otimes I - (|01\rangle\langle 01| + |10\rangle\langle 10|)\otimes I$$

1，如果同.　　　　　　−1，若不同.

Three qubit phase flip code

another noisy quantum channel

phase flip error model for a single qubit

phase flip $\longrightarrow$ bit flip ?

In qubit basis $|+\rangle, |-\rangle$. $\quad$ Z takes $|+\rangle$ to $|-\rangle$.

bit flip with respect to the labels + and −

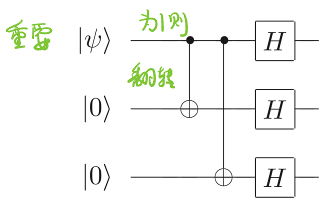


Figure 10.3. Encoding circuits for the phase flip code.

$|\psi\rangle = a|0\rangle + b|1\rangle$.

为$|0\rangle$时，$|000\rangle$.

$|1\rangle$时. 因为翻转了，因此$|111\rangle$.

Error detection:

Projective measurements conjugated by H gates:

$$P_j \longrightarrow P_j' \equiv H^{\otimes 3} P_j H^{\otimes 3}$$

4.1 Nonseperability of EPR pairs

△ Hidden quantum information

A bipartite pure state is entangled if its Schmidt number >1.

maximally entangled state of two qubits (EPR pair)

$$|\phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + |11\rangle_{AB}\right)$$

→ meaning: $\rho_A = \mathrm{Tr}_B(|\phi^+\rangle\langle\phi^+|) = \frac{1}{2}I_A$.

measure spin A along any axis result is completely random.

⇒ if perform any local measurement of A or B, acquire no info about preparation of state. generate random bit.

(contrast: single qubit in pure state. can store a bit by preparing $|\uparrow_{\hat{n}}\rangle$ or $|\downarrow_{\hat{n}}\rangle$)

two qubits ⟶ store two bits?

but in $|\phi^+\rangle$ this info is hidden

Bell basis

$$|\phi^\pm\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right).$$

$$|\psi^\pm\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right).$$

不是很奇怪吗？$|00\rangle$是一种态，$|11\rangle$也是一种、却把这包含2个spin的2个态都考虑进了一个态中。如何做到这种提取和包含的？

Charlie prepare one of these four states. encoding two bits:

1. Parity bit ($|\phi\rangle$ or $|\psi\rangle$) 2. Phase bit ($|+\rangle$ or $|-\rangle$).

Alice ⟵ qubit A   qubit B ⟶ Bob

local manipulation.

flip relative phase $\vec{\sigma}_3$: $|\phi^+\rangle \to |\phi^-\rangle.$

$|0\rangle + |1\rangle$ $\quad |\psi^+\rangle \to |\psi^-\rangle.$

$\hookrightarrow |0\rangle - |1\rangle$

flip parity $\vec{\sigma}_1$: $|\phi^+\rangle \to |\psi^+\rangle$

$|0\rangle \leftrightarrow |1\rangle$ $\quad |\phi^-\rangle \to -|\psi^-\rangle.$

△ Suppose Alice & Bob able to exchange classical info

entangled basis states: simultaneous eigenstates of two commuting observables

$$\sigma_1^{(A)} \otimes \sigma_1^{(B)} \quad , \quad \sigma_3^{(A)} \otimes \sigma_3^{(B)}$$

(eigenvalue) phase bit parity bit

Alice & Bob measure along z-axis. prepare a simultaneous eigenstate of $\sigma_3^{(A)}$ and $\sigma_3^{(B)}$. ($\sigma_3^{(A)}$, $\sigma_3^{(B)}$, $\sigma_3^{(A)} \otimes \sigma_3^{(B)}$ commute. will not disturb parity bit)

But $\sigma_1^{(A)} \otimes \sigma_1^{(B)}$, $\boxed{\sigma_3^{(A)}, \sigma_3^{(B)}}$ not commute, will disturb phase bit.

How to measure without disturbing ?

Should learn $\sigma_3^{(A)} \otimes \sigma_3^{(B)}$ without knowing $\sigma_3^{(A)}$ or $\sigma_3^{(B)}$.

△ Now bring Alice & Bob together. (operate on qubits jointly)

How they acquire both parity & phase bit.

Apply unitary operation so entangled basis $\{|\phi^\pm\rangle, |\psi^\pm\rangle\} \Rightarrow$ unentangled $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$.

then they can measure A . B seperately.

A particular single-qubit unitary :

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}}(\sigma_1 + \sigma_3) . \quad H^2 = I. \quad H\sigma_1 H = \sigma_3, \quad H\sigma_3 H = \sigma_1 .$$

A $\theta = \pi$ rotation about $\hat{n} = \frac{1}{\sqrt{2}}(\hat{n}_1 + \hat{n}_3)$. $\hat{x}$ to $\hat{z}$.

$$U(\hat{n}, \theta) = I\cos\frac{\theta}{2} + i\vec{n}\cdot\vec{\sigma}\sin\frac{\theta}{2} = i\frac{1}{\sqrt{2}}(\vec{\sigma}_1 + \vec{\sigma}_3) = iH.$$

A two-qubit transformation. reversible XOR or controlled-NOT transformation

Why cannot create distant entanglement?

Of course, $\boldsymbol{H}$ acts on only one of the qubits; the "nonlocal" part of our circuit is the controlled-NOT gate — this is the operation that establishes or removes entanglement. If we could only perform an "interstellar **CNOT**," we would be able to create entanglement among distantly separated pairs, or extract the information encoded in entanglement. But we can't. To do its job, the **CNOT** gate must act on its target without revealing the value of its source. Local operations and classical communication will not suffice.

置换对称性

粒子不可区分.

$|k'\rangle|k''\rangle$ $\qquad$ $|k''\rangle|k'\rangle$ $\qquad$ 而 k', k'' 是可区分的右矢.

所有具有 $c_1|k'\rangle|k''\rangle + c_2|k''\rangle|k'\rangle$ 形式的右矢

将导致完全相同的本征值集合. $\Longrightarrow$ 交换简并.

困难: 一个可观测量完备集本征值的规定不能完全确定这个态右矢.

$$A_1|a'\rangle|a''\rangle = a'|a'\rangle|a''\rangle \qquad P_{12}A_1P_{12}^{-1}\underbrace{P_{12}|a'\rangle|a''\rangle}_{|a''\rangle|a'\rangle} = a'\underbrace{P_{12}|a'\rangle|a''\rangle}_{|a''\rangle|a'\rangle}$$

$$A_2|a'\rangle|a''\rangle = a''|a'\rangle|a''\rangle$$

仅当 $P_{12}A_1P_{12}^{-1} = A_2$ 时一致. $\qquad$ P_{12} 一定会交换可观测量的粒子标号.

P_{12} 与 H 对易. 运动常数. $\qquad$ 开始对称(反对称)则保持不变.

二次量子化

多粒子态矢量 $|n_1, n_2, \cdots n_i, \cdots\rangle$ Fock space

(为何这样选态矢?)

隐含假设. 存在一组无相互作用的基.

$|0,0,\cdots 0\cdots\rangle = |\vec{0}\rangle$. 真空.

$|0,0,\cdots n_i=1,\cdots\rangle \equiv |k_i\rangle$. 单粒子态.

定义场算符 $a_i^\dagger$.

$$a_i^\dagger |n_1, n_2, \cdots n_i, \cdots\rangle \propto |n_1, n_2, \cdots n_i+1, \cdots\rangle$$

$$a_i^\dagger |\vec{0}\rangle = |k_i\rangle. \qquad 1 = \langle k_i | k_i\rangle = \langle \vec{0} | a_i a_i^\dagger |\vec{0}\rangle = \langle 0 | \underbrace{a_i | k_i\rangle}_{\text{湮灭}} = 0$$

$$a_i |k_j\rangle = \delta_{ij} |\vec{0}\rangle.$$

引入交换对称性. $|k_i\rangle$ $|k_j\rangle$

对两粒子态 $a_i^\dagger a_j^\dagger |\vec{0}\rangle = \pm a_j^\dagger a_i^\dagger |\vec{0}\rangle.$

$$a_i^\dagger a_j^\dagger - a_j^\dagger a_i^\dagger = [a_i^\dagger, a_j^\dagger] = 0. \quad \text{Boson} \quad \xrightarrow{\text{取共轭}} \quad [a_i, a_j] = 0$$

$$\underbrace{a_i^\dagger a_j^\dagger + a_j^\dagger a_i^\dagger}_{\Downarrow\ a_i^\dagger a_i^\dagger = 0} = \{a_i^\dagger, a_j^\dagger\} = 0. \quad \text{Fermion} \qquad \{a_i^\dagger, a_j^\dagger\} = 0$$

$a_i^\dagger a_i^\dagger = 0$. 包含 Pauli 不相容原理

a_i 与 $a_j^\dagger$ 的对易关系? 定义 $N_i = a_i^\dagger a_i$. 对处于单粒子态 $|k_i\rangle$ 粒子数计数.

Boson	Fermion
$a_i^\dagger a_j^\dagger - a_j^\dagger a_i^\dagger = [a_i^\dagger, a_j^\dagger] = 0$	$a_i^\dagger a_j^\dagger + a_j^\dagger a_i^\dagger = \{a_i^\dagger, a_j^\dagger\} = 0$
$a_i a_j - a_j a_i = [a_i, a_j] = 0$	$a_i a_j + a_j a_i = \{a_i, a_j\} = 0$
$a_i a_j^\dagger - a_j^\dagger a_i = [a_i, a_j^\dagger] = \delta_{ij}$	$a_i a_j^\dagger + a_j^\dagger a_i = \{a_i, a_j^\dagger\} = 0.$

多费米系统

互换产生负相移

由 Pauli 不相容原理 和只存在跃迁项的 H 刻画.

可由反对易算符描述

△ 晶格上的无自旋费米系统

H 基矢 $\{|n_{i_1}, n_{i_2}, \cdots\rangle\}$. $n_i = 0, 1$, 格点 i 的费米子数

为每个格点引入

$$\sigma_i^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \frac{1}{2}(\sigma_x - i\sigma_y).$$ 湮灭

玻色子算符?

$$\sigma_i^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \frac{1}{2}(\sigma_x + i\sigma_y).$$ 生成

Fermi 系统的 Hamiltonian:

实际上是硬核 Boson 系统

或 spin-1/2 系统

$$H = \sum_{\langle ij\rangle} \left(t_{ij}\sigma_i^+\sigma_j^- + h.c.\right).$$

△ 满足 Pauli 不相容原理的 Fermi Hilbert 空间

△ 特别的 Hamiltonian

$$H_f = \sum_{\langle ij\rangle} \left[t_{ij}(\{\sigma_i^z\})\,\sigma_i^+\sigma_j^- + h.c.\right].$$

简化. 排序格点: $(i_1, i_2, \cdots i_a, \cdots)$

$$C_{i_a} = \sigma_{i_a}^- \prod_{b<a} \sigma_{i_b}^z.$$

$$\{C_i, C_j\} = \{C_i^+, C_j^+\} = 0$$

$$\{C_i, C_j^+\} = \delta_{ij}.$$

$$H_f = \sum_{\langle ij\rangle} \left(t_{ij} C_i^+ C_j + h.c.\right).$$

$\sigma^{x,y,z} \to C_i$. Jordan - Wigner Transformation

Fermion 是非局域激发.

C_{2j-1}, C_{2j} different site

phase error $a_j^{\dagger} a_j = \frac{1}{2}(1 + i C_{2j-1} C_{2j})$

$$H = H_0 + H'.$$

$$H_0 = \sum_{i=1}^{N-1} K(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+)$$

$g \ll K$ H' perturbation

$$H' = g(S_0^+ S_1^- + S_{N+1}^+ S_N^- + h.c.)$$

introduce fermi operator

$$C_i = e^{i\pi \sum_{0}^{i-1} S_j^+ S_j^-} S_i^- .$$

$$H_0 \Rightarrow \sum_{i=1}^{N-1} K(C_i^+ C_{i+1} + C_i C_{i+1}^+).$$

conservation of S_z $\rightarrow$ conservation of fermion number

orthogonal transformation to diagonalize

$$f_k^+ = \frac{1}{A} \sum_{j=1}^{N} \sin \frac{jk\pi}{N+1} C_j^+ .$$

$$A = \sqrt{\frac{N+1}{2}} .$$

$$\Rightarrow H_0 = \sum_{k=1}^{N} E_k f_k^+ f_k . \qquad E_k = 2K \cos \frac{k\pi}{N+1} .$$

$$\Rightarrow H' = \sum_{k=1}^{N} t_k \left(C_0^+ f_k + (-1)^{k-1} C_{N+1}^+ f_k + h.c.\right) \qquad t_k = \frac{g}{A} \sin \frac{k\pi}{N+1} .$$

odd N. exists a single zero energy fermionic mode $k = z \equiv \frac{N+1}{2}$.

two end spins resonantly coupled to zero energy fermion by H'.

assumption $t_z \sim g/A \ll$ fermion detuning $|E_z - E_{z\pm1}| \sim K/N$.

other modes can be neglected.

Upon absorbing phase factor $(-1)^{z-1}$ into C_{N+1}.

$$H_{eff} = t_z(C_0^+ f_z + C_{N+1}^+ f_z + h.c.)$$

resonant fermonic tunneling

Unitary evolution under $\tau = \frac{\pi}{\sqrt{2} t_z}$.

$$U_{eff} = e^{-i\tau H_{eff}} = (-1)^{f_z^\dagger f_z} \left(1 - (c_0^\dagger + c_{N+1}^\dagger)\right)(c_0 + c_{N+1}).$$

Upon projection to subspace $\{(1, c_0^\dagger, c_{N+1}^\dagger, c_0^\dagger c_{N+1}^\dagger)|00\rangle_{0,N+1}\}$

the effective resolution

$$U_{eff}^{fermi} = (-1)^{n_0 + n_{N+1} + n_z} (-1)^{n_0 n_{N+1}} \, SWAP_{0,N+1}$$

entangled through a CPHASE gate. $CP_{0,N+1} = (-1)^{n_0 n_{N+1}}$.

$$S_i^+ S_j^- + S_i^- S_j^+$$

$$(S_{ix} + iS_{iy}) \cdot (S_{jx} - iS_{jy}) + (S_{ix} - iS_{iy}) \cdot (S_{jx} + iS_{jy})$$

$$(S_{ix} S_{jx} + S_{iy} S_{jy}) \longrightarrow \vec{S}_i \cdot \vec{S}_j$$

$\underline{c_i^\dagger c_{i+1} + c_i c_{i+1}^\dagger}$? derive

$$H_G = -\sum_{\langle i,j\rangle} J_{ij}\,\vec{\sigma}_i\cdot\vec{\sigma}_j - \sum_{i=1}^{N} B_i\,\sigma_z^i. \qquad [H_G, \sigma_z] = 0$$

arbitrary FM with isotropic Heisenberg interactions

Initialize to ground state $|\vec{0}\rangle = |00\cdots0\rangle$

easy for FM system by cooling. set $E_0 = 0$.

$|\vec{j}\rangle = |0\cdots010\cdots0\rangle$.

channel spins

$\vec{j} = 1, 2, \cdots s, \ldots r, \cdots N$.

Alice

sender

Bob

receiver

$t=0$. sth spin. $|\psi_{in}\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\theta}\sin\frac{\theta}{2}|1\rangle$.

State of whole chain $|\psi(0)\rangle = \cos\frac{\theta}{2}|\vec{0}\rangle + e^{i\phi}\sin\frac{\theta}{2}|\vec{s}\rangle$.

Bob wait for $|\psi(0)\rangle$ to involve to a final state

as close as possible to $\cos\frac{\theta}{2}|\vec{0}\rangle + e^{i\phi}\sin\frac{\theta}{2}|\vec{r}\rangle$.

$$|\psi(t)\rangle = \cos\frac{\theta}{2}|\vec{0}\rangle + e^{i\varphi}\sin\frac{\theta}{2}\sum_{j=1}^{N}\langle j|e^{-iH_G t}|s\rangle|j\rangle$$

?

output state $\mathrm{Tr}_{1,2\cdots N-1}\left(|\psi(t)\rangle\langle\psi(t)|\right)$

$$\rho_{out} = P(t)\,|\psi_{out}(t)\rangle\langle\psi_{out}(t)| + \left(1-P(t)\right)|0\rangle\langle 0|$$

$e^{-iHt}|\psi\rangle$ for t-independent H_G. ✓

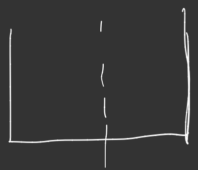
$$e^{-iHt}|\vec{r}\rangle = ?$$

懂了. $|\vec{s}\rangle$ 只变成 $|\vec{j}\rangle$ 意思是 spin 个守恒. --

$$\underbrace{\sum_{j=1}^{N}|j\rangle\langle j|}\,e^{-iH_G t}|s\rangle \quad ✓$$

Assume $J_{ij} = J/2\,\delta_{i+1,j}$. $B_i = B$.

eigenstates $$|\tilde{m}\rangle_L = a_m\sum_{j=1}^{N}\cos\left\{\frac{\pi}{2N}(m-1)(2j-1)\right\}|\vec{j}\rangle.$$



$x=0$

$$\phi_k(-x) = (-1)^k \phi_k(x).$$

alternating parity

$$\psi(x,t) = \sum_k c_k e^{-iE_k t} \phi_k(x). \qquad E_k \sim k^2.$$

commensurate energy

If we choose $t=\tau$. $\quad \tau = \dfrac{k^2\pi}{E_k}.$

mirror inverts

$$\psi(x,\tau) = \sum_k c_k (-1)^k \phi_k(x) = \sum_k c_k \phi_k(-x) = \psi(-x).$$

analogy [position of particle in a line
single spin flip in a background of aligned spins

$x \qquad \psi(x)$

$j=1,\cdots N \qquad \sum_j c_j |j\rangle.$

If $J_{j,j+1} = J_{N-j,N-j+1}$.

LAS VEGAS

LINQ

Fermi liquid interaction

e-p coupling

stacked flat bands

graphene superlattice potential

topological state under decoherence

2D cluster state SPT

which aspect of teleportation?

$\gamma = \sum u_i c_i + v_i c_i^\dagger$

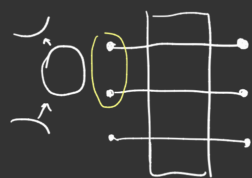
u_i, v_i

same state at different location ---

eject a fermion

not ... happen

STM

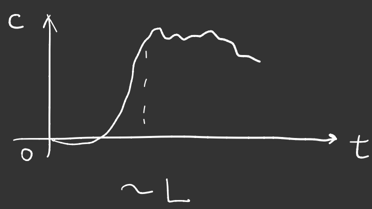


correlation function

n_1 n_2

$\langle n_2(t)\, n_1(0)\rangle \to \langle n_2(t)\rangle\langle n_1(0)\rangle$

XX+YY



$S_x S_x$ $h S_z$

Ising

$t c_i c_{i+1}^\dagger + \Delta c_i c_{i+1}^\dagger$ $\sqrt{t^2 \cos^2 k + \Delta^2 \sin^2 k}$ $E \to 0$

epr-like setup using majorana wire

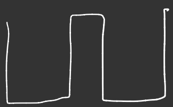
0601261

$|\psi\rangle$

state described by wavefunction well-seperated

↪ eigenenergy seperated

V

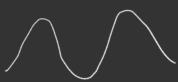


create a particle in ground state ?

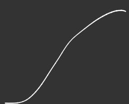
populate groud state ✗

→ place the particle into a superpostion of symmetric & anti-symmetric states

→ tunnel to symmetrize its state



ground state



antisymmetric state ?

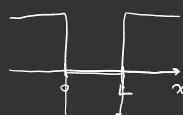
(close to degenerate)

"isolated" Single-particle states ?

— Dirac equation (interacting with topologically non-trivial backgrounds)

1D. $[i\gamma^\mu \partial_\mu + \phi(x)]\psi(x,t) = 0.$

$$\phi(x) = \begin{cases} \phi_0 & x<0,\ x>L \\ -\phi_0 & 0<x<L. \end{cases}$$



$$i\begin{pmatrix} & \frac{d}{dx}+\phi(x) \\ \frac{d}{dx}-\phi(x) & \end{pmatrix}\begin{pmatrix} u_E(x) \\ v_E(x) \end{pmatrix} = E\begin{pmatrix} u_E(x) \\ v_E(x) \end{pmatrix}$$

two bound states

$$E_+ \approx \phi_0 e^{-\phi_0 L}. \quad \psi_+ \approx \sqrt{\phi_0}\begin{pmatrix} e^{-\phi_0|x|} \\ -ie^{-\phi_0|L-x|} \end{pmatrix} + O(e^{-\phi_0 L})$$

$$E_- \approx -\phi_0 e^{-\phi_0 L}. \quad \psi_- \approx \sqrt{\phi_0}\begin{pmatrix} e^{-\phi_0|x|} \\ ie^{-\phi_0|L-x|} \end{pmatrix} + O(e^{-\phi_0 L})$$

many fermion system. E_- filled. E_+ empty.

fermion / anti-fermion excited by

create a fermion by populating the positive energy bound state. has ψ_+. 

anti-fermion ψ_- degenerate ..?

$\phi_0 L$ large. $E_+, E_- \to 0$.

$$\psi_0 = \frac{1}{\sqrt{2}}\left(e^{iE_0 t}\psi_+ + e^{-iE_0 t}\psi_-\right) \approx \sqrt{2\phi_0}\begin{pmatrix} e^{-\phi_0|x|}\cos E_0 t \\ e^{-\phi_0|L-x|}\sin E_0 t \end{pmatrix} + O(e^{-\phi_0 L})$$

for $t \ll \frac{1}{E_0}$. around $x=0$.

$$\psi_L = \frac{1}{\sqrt{2}i}\left(e^{iE_0 t}\psi_+ - e^{-iE_0 t}\psi_-\right) \approx \sqrt{2\phi_0}\begin{pmatrix} e^{-\phi_0|x|}\sin E_0 t \\ -e^{-\phi_0|L-x|}\cos E_0 t \end{pmatrix} + O(e^{-\phi_0 L})$$

near $x=L$.

relevant state could be anything but the ground state that has

$\psi_-(x)$ populated and $\psi_+(x)$ empty

dump fermion $x=0$ $\Rightarrow$ populate ψ_0 rather than ψ_+

degenerate fermion/anti-fermion state

Majorana. better.

Hamiltonian of majorana. map + energy to − energy states

$$\psi_{-E}(x) = \psi_E^*(x).$$

fermion, anti-fermion $\longrightarrow$ majorana fermion number not conserved (?) parity conserved.

Majorana fermion. only one bound state. wavefunction ψ_+. occupy or empty.

$(-1)^F = -1$. occupy. $(-1)^F = 1$. empty

ψ_0. ψ_L. do not have definite fermion parity.

begin with system with quantum state an eigenstate of fermion parity

Hard? decompose. rapidly remixed by EM interactions.

SC. decomposition more efficiently. charge screened.

Bogoliubov quasi-fermions can be majorana fermions when the

SC condensate is parity-odd. (p-wave e.g.)

mid-gap bound states. — Andreev states (at surface)

p-wave SC. giving vortices non-Abelian fractional statistics

weak coupling to SC

— electrons can enter and leave the wire

as Cooper pairs

$$H = \sum_{n=1}^{L} \left(\frac{t}{2} a_{n+1}^{\dagger} a_n + \frac{t^*}{2} a_n^{\dagger} a_{n+1} + \frac{\Delta}{2} a_{n+1}^{\dagger} a_n^{\dagger} + \frac{\Delta^*}{2} a_n a_{n+1} + \mu a_n^{\dagger} a_n \right)$$

$|\mu| < |t|$. $|\Delta| < |t|$.

$t = |t| e^{i\theta}$. $\Delta = |\Delta| e^{2i\theta}$.

$a_n \longrightarrow e^{i(\phi+\theta)} a_n$ odd n

$a_n \longrightarrow e^{i(\phi-\theta)} a_n$ even n

$$i\frac{d}{dt} a_n = \frac{t}{2}(a_{n+1}+a_{n-1}) - \frac{\Delta}{2}(a_{n+1}^{\dagger} - a_{n-1}^{\dagger}) + \mu a_n .$$

decompose $a_n = b_n + i c_n$. $\psi_n = \begin{pmatrix} b_n \\ c_n \end{pmatrix}$. $\psi_n(t) = e^{i\omega t}\psi_n$.

$$\psi_{n+2} = -N\psi_{n+1} - M\psi_n .$$

$$\begin{pmatrix} 2\mu & \frac{2i\omega}{t-\Delta} \\ \frac{-2i\omega}{t+\Delta} & \frac{2\mu}{t+\Delta} \end{pmatrix} \qquad \begin{pmatrix} \frac{t+\Delta}{t-\Delta} & \\ & \frac{t-\Delta}{t+\Delta} \end{pmatrix}$$

1512.03026

adiabatic transfer protocols → STIRAP stimulated Raman adiabatic passage

coherent tunneling by adiabatic passage

vs

resonant techniques

robust against pulse area & timing errors

useful when interact via lossy intermediate

con: slow, suffer from dissipation / noise in source / target

speed up? counteradiabatic control (transitionless driving)

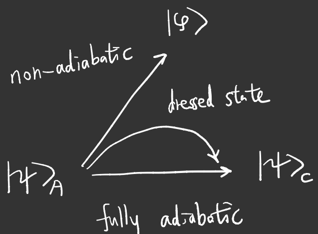
modification of H_0 to compensate for nonadiabatic errors

In principle transitionless driving would allow a perfect state transfer

but: require direct coupling of S/T, or coupling not available in H_0.

constructing dynamical invariants? optimal quantum control?

allow for perfect state transfer even in nonadiabatic regime



develop simple and effective pulses for speeding up adiabatic state transfer in Λ-system

construct $H(t)$. instantaneous (adiabatic) eigenstates

$$\hat{H}(t)|\varphi_k(t)\rangle = E_k(t)|\varphi_k(t)\rangle.$$

$\{|\varphi_{mj}(t)\rangle\}_{j=0}^{n}$ serve as "medium" states.

$$|\varphi_{mj}(t_i)\rangle = |\beta_j\rangle_A \otimes |\chi_i\rangle_{B,C}$$

$$|\varphi_{mj}(t_j)\rangle = |\chi_f\rangle_{A,B} \otimes |\gamma_j\rangle_C$$

if the evolution is perfectly adiabatic ($\tau \gg 1/\Delta E$)

move to adiabatic frame (adiabatic eigenstates are time-independent)

$$\hat{U}(t) = \sum_k |\varphi_k\rangle\langle\varphi_k(t)|.$$

$$\hat{H}_{ad}(t) = \hat{H}_0(t) + \underbrace{\hat{W}(t)} = \sum_k E_k(t)|\varphi_k\rangle\langle\varphi_k| + i\frac{d\hat{U}(t)}{dt}\hat{U}^\dagger(t).$$

generically have off-diagonal matrix elements

magnitude ↑ as τ ↓. ⇒ imperfect state transfer

correcting nonadiabatic errors

$\underbrace{\hat{H}_{mod}(t)} = \hat{H}(t) + \underbrace{\hat{H}_c(t)}$. does not involve couplings that cannot be experimentally implemented

no attainably large coupling strength

△ the corrected dynamics only needs to evolve the system from correct state t_i to t_f

I) a new basis of dressed states $|\tilde{\varphi}_k(t)\rangle$ formally defined by $V(t)$

$$|\tilde{\varphi}_k(t)\rangle = \hat{V}^\dagger(t)|\varphi_k\rangle.$$

II) a control field $\hat{H}_c(t)$ added

Constraints:

△ $\hat{V}^\dagger(t_f)|\varphi_{m_j}\rangle = \hat{V}^\dagger(t_i)|\varphi_{m_j}\rangle = |\varphi_{m_j}\rangle$.

△ for all j. evolution of $|\varphi_{m_j}(t)\rangle$ is trivial in the basis defined by $\hat{V}(t)$. $|\tilde{\varphi}_{m_j}(t)\rangle\langle\tilde{\varphi}_{m_j}(t)|$ conserved.

$$\hat{H}_{new}(t) = \hat{V}\hat{H}_{ad}(t)\hat{V}^\dagger + \hat{V}\hat{U}\hat{H}_c(t)\hat{U}^\dagger\hat{V}^\dagger + i\frac{d\hat{V}}{dt}\hat{V}^\dagger.$$

$\langle\tilde{\varphi}_{m_j}|\hat{H}_{new}|\tilde{\varphi}_k\rangle = 0$. for $1 \le k \le n$. $k \ne m_j$. (H_c cancel!!)

special case: transitionless driving. $\hat{V}(t) = \mathbb{1}$. $\hat{H}_c = -\hat{U}^\dagger\hat{W}\hat{U}$.

STIRAP. $A \rightleftarrows B \rightarrow C$

adiabatic eigenstates

two bright states $|\varphi_\pm(t)\rangle$. $E_\pm(t) = \pm\Omega(t)$.

dark state. $|\varphi_D(t)\rangle$. $E_D = 0$.

$$H(t) = \underset{\text{pump}}{\Omega_p(t)} |B\rangle\langle A| + \underset{\text{stokes}}{\Omega_s(t)} |B\rangle\langle C| + h.c.$$

$$|\varphi_D(t)\rangle = \cos\theta(t)|A\rangle + \sin\theta(t)|C\rangle.$$

$$|\varphi_0(t)\rangle = |000\rangle. \quad E_0(t) = 0.$$

$$U_{ad} = \begin{array}{cc} \begin{pmatrix} \sin\theta(t)/\sqrt{2} & -1/\sqrt{2} & -\cos\theta(t)/\sqrt{2} \\ \cos\theta(t) & 0 & \sin\theta(t) \\ \sin\theta(t)/\sqrt{2} & 1/\sqrt{2} & -\cos\theta(t)/\sqrt{2} \end{pmatrix} & \begin{matrix} |\varphi_+\rangle \\ |\varphi_D\rangle \\ |\varphi_-\rangle \end{matrix} \end{array}$$

$\theta(t_i) = 0,$

$\theta(t_f) = \pi/2.$

protocol time reduced. more corrupted.

$$\hat{H}_{ad}(t) = \Omega(t)\hat{M}_z + \underbrace{\dot{\theta}(t)\hat{M}_y}_{\text{non-adiabatic couplings}}.$$

$$\hat{M}_z = |\varphi_+\rangle\langle\varphi_+| - |\varphi_-\rangle\langle\varphi_-|.$$

$$\hat{M}_x = (|\varphi_-\rangle - |\varphi_+\rangle)\langle\varphi_D|/\sqrt{2} + h.c.$$

$$[M_p, M_q] = i\varepsilon^{pqr} M_r.$$

$$g_x(t) = \frac{\dot{\mu}}{\cos\xi} - \dot{\theta}\tan\xi.$$

$$g_z(t) = -\Omega + \dot{\xi} + \frac{\dot{\mu}\sin\xi - \dot{\theta}}{\tan\mu\cos\xi}.$$

$$H_{new} = -\frac{\dot{\theta} + \dot{\xi} + \dot{\mu}\sin\xi}{\sin\mu\cos\xi}\hat{M}_z$$

$$= \tilde{E}(t)\hat{M}_z.$$

$$|\psi(t)\rangle = U_{ad}^\dagger V_g^\dagger(t)\exp\left(-i\int_{t_i}^{t} dt'\,\tilde{E}(t')\hat{M}_z\right) V_g(t_i)\,U_{ad}(t_i)\,|\psi(t_i)\rangle.$$

topological SC. nontrivial. distinct from ordinary SC and vacuum.

↳ phase. must exhibit localized modes at boundaries.

Dirac matrix. (Majorana representation). real field solution. zero-energy stationary state solution

↳ U(1) gauge symmetry ✗. lepton not conserved. charge neutral. spin 1/2.

Fermi liquid. $\dfrac{\text{fermi surface} \qquad \text{electronic excitation}}{\text{hole-type excitation}}$ > quasi-anti-particle ?

⇒ Bogliubov quasi-particle in SC ?

s-wave SC. $H^s_{pair} = \sum_k \Delta_s c_{\vec{k},\uparrow} c_{-\vec{k},\downarrow} + h.c.$

$\xrightarrow[\text{(electron-hole representation)}]{\text{Nambu space}}$ $H_{pair} = \sum_k \Delta_s d^\dagger_{\vec{k},\uparrow} c_{\vec{k},\downarrow} + h.c.$ $d_{\vec{k},\uparrow} = c^\dagger_{-\vec{k},\uparrow}$ hole operator

excitation in s-wave SC. $b_{\vec{k}} = u c_{\vec{k},\uparrow} + v d_{\vec{k},\downarrow} = \underbrace{u c_{\vec{k},\uparrow} + v c^\dagger_{-\vec{k},\downarrow}}$. not self-conjugate

⇒ SC formed with identical spin

odd parity

p-wave SC. $H^p_{pair} = \sum_p \Delta_p(\vec{k}) c_{\vec{k}} c_{-\vec{k}} + h.c.$ $\Delta_p(\vec{k}) = -\Delta_p(-\vec{k})$.

excitation $\gamma_{\vec{k}} = u c_{\vec{k}} + v c_{\vec{k}}$. $\xrightarrow{\text{real space}}$ $\gamma(\vec{x}) = u(\vec{x}) c(\vec{x}) + v(\vec{x}) c^\dagger(\vec{x})$. $u = v^*$. majorana.

MZM. in 1D p-wave TSC. boundary. 2D p+ip TSC. vortice center.

self-conjugate. do not have number space. $\gamma^\dagger \gamma = \gamma^2 = 1$.

2 MZM → Dirac fermion. $|0\rangle . |1\rangle$ MZM. quantum dimension $\sqrt{2}$. half complex fermion.

non-Abelian statistics

QI stored in a non-local form

1D spinless p-wave SC chain

$\{c_i, c_j\} = \{c_i^+, c_j^+\} = 0 \quad i \neq j$

$\{c_i, c_j^+\} = \delta_{ij}$.

$$H = -\mu \sum_{k=1}^{N} c_k^+ c_k - \sum_{k=1}^{N-1} (t c_k^+ c_k + \underbrace{\Delta}_{\text{nearest-neighbour pairing strength}} \overbrace{e^{i\phi}}^{\text{SC pairing phase}} c_k c_k + h.c.)$$

$c_i c_j + c_j c_i = 0$

$c_i c_j^+ + c_j^+ c_i = 0. \ (i \neq j)$

$c_i c_i^+ + c_i^+ c_i = 1$.

$\gamma_{k,A} = i(c_k^+ e^{-i\phi/2} - c_k e^{i\phi/2})$.

$\gamma_{k,B} = c_k^+ e^{-i\phi/2} + c_k e^{i\phi/2}$.

$\{\gamma_{\alpha,k}, \gamma_{\alpha',k'}\} = 2\delta_{\alpha\alpha'}\delta_{kk'}$.

$$H = -\frac{\mu}{2} \sum_{k=1}^{N} (1 + i\gamma_{k,B}\gamma_{k,A}) - \frac{i}{4} \sum_{k-1}^{N-1} [(\Delta+t)\gamma_{k,B}\gamma_{k+1,A} + (\Delta-t)\gamma_{k,A}\gamma_{k+1,B}].$$

$c_k e^{i\phi/2} = (\gamma_{k,B} + i\gamma_{k,A})/2$

$c_k^+ e^{-i\phi/2} = (\gamma_{k,B} - i\gamma_{k,A})/2$

$\gamma_{k,A}^2 + \gamma_{k,B}^2 = 2(c_k^+ c_k + c_k c_k^+)$

$(\gamma_{k,B} - i\gamma_{k,A})(\gamma_{k,B} + i\gamma_{k,A})$

$c_k^+ c_k = \gamma_{k,A}^2 + \gamma_{k,B}^2 - i\gamma_{k,A}\gamma_{k,B} + i\gamma_{k,B}\gamma_{k,A}$

$\{\gamma_{k,A}, \gamma_{k,B}\} = \gamma_{k,A}\gamma_{k,B} + \gamma_{k,B}\gamma_{k,A} = 0$

$i\gamma_{k,B}\gamma_{k,A} = -1(c_k^+ c_k^+ e^{-i\phi} - c_k^+ c_k + c_k c_k^+ - c_k c_k e^{i\phi})$

$= i(c_k^+ c_k^+ e^{-i\phi} - c_k c_k^+ + c_k^+ c_k - c_k c_k e^{i\phi})$

$+ i(c_k^+ c_k^+ e^{-i\phi} + c_k c_k^+ - c_k^+ c_k - c_k c_k e^{i\phi})$

?

$= 2i(c_k^+ c_k^+ e^{-i\phi} - c_k c_k e^{i\phi})$?

$$H_1 = \sum_j \Big[-t\underbrace{(a_j^+ a_{j+1} + a_{j+1}^+ a_j)}_{\neq 0 \text{ for periodic conditions}} - \mu(a_j^+ a_j - \tfrac{1}{2}) + \Delta a_j a_{j+1} + \Delta^* a_{j+1}^+ a_j^+ \Big].$$

$\Delta = |\Delta| e^{i\theta}$.

$c_{2j-1} = e^{i\frac{\theta}{2}} a_j + e^{-i\frac{\theta}{2}} a_j^+$

$a_j = (c_{2j-1} + ic_{2j})/2 \cdot e^{-i\frac{\theta}{2}}$

$c_{2j} = -i(e^{i\frac{\theta}{2}} a_j - e^{-i\frac{\theta}{2}} a_j^+)$.

$a_j^+ = (c_{2j-1} - ic_{2j})/2 \cdot e^{i\frac{\theta}{2}}$

$$a_j^+ a_{j+1} + a_{j+1}^+ a_j = \frac{c_{2j-1} - ic_{2j}}{2} \cdot \frac{c_{2j+1} + ic_{2j+2}}{2} + \frac{c_{2j+1} - ic_{2j+2}}{2} \cdot \frac{c_{2j-1} + ic_{2j}}{2}$$

$c_{2j-1}^+ = e^{-i\frac{\theta}{2}} a_j^+ + e^{i\frac{\theta}{2}} a_j = c_{2j-1}$. ✓

$$= \Big(\underbrace{c_{2j-1} c_{2j+1}}_{\text{II}\,0} - i c_{2j} c_{2j+1} + i c_{2j-1} c_{2j+2} + \underbrace{c_{2j} c_{2j+2}}_{\text{II}\,0}$$

$c_{2j-1}^2 = c_{2j-1}^{+2} = 1$?

$\{c_i, c_j\} = 2\delta_{ij}$?

$c_j^+ = c_j$

$$+ c_{2j+1} c_{2j-1} - i c_{2j+2} c_{2j-1} + i c_{2j+1} c_{2j} + c_{2j+2} c_{2j} \Big) \cdot \frac{1}{4}$$

$c_i c_j + c_j c_i = 0$

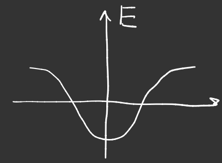
$c_j^2 = c_j^+ c_j = c_j c_j^+ = 1$?

$$= \frac{i}{2}\left(C_{2j-1}C_{2j+2} + C_{2j+1}C_{2j}\right)$$

推一遍?

$$\cdots \quad H_1 = \frac{i}{2}\sum_j \left(-\mu C_{2j-1}C_{2j} + (\omega+|\Delta|)\, C_{2j}C_{2j+1} + (-\omega+|\Delta|)\, C_{2j-1}C_{2j+2}\right).$$

$\mu \neq 0$. $\Delta = t = 0$. $H \dashrightarrow -\mu \sum_j C_j^\dagger C_j$. $a_j a_j$



couple. $\Rightarrow$ fermionic excitation

no MZM. no zero-energy excitation.

$\mu = 0$. $\Delta = t$. couple through Δ, t.

$\gamma_{1,A}, \gamma_{N,B}$ not shown in Hamiltonian!

MZM on two ends.

$f = \frac{1}{2}(\gamma_{1,A} + i\gamma_{N,B})$. zero excitation energy?

why? 2-fold degenerate ground state $|0\rangle$. $f^\dagger|0\rangle = |1\rangle$.

$H = \frac{1}{2}\sum_k C_k^\dagger H(k) C_k$. $C_k = (C_k, C_{-k}^\dagger)$. Nambu space

真的是完全不同的拓扑相吗?

$$H(k) = \begin{pmatrix} -t\cos k - \mu & i\Delta e^{-i\phi}\sin k \\ -i\Delta e^{i\phi}\sin k & t\cos k + \mu \end{pmatrix}.$$

topological invariant. 1D winding number?

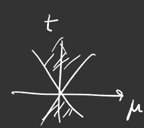
k change 2π.

$\phi = 0$. $H(k) = h_z\tau_z + h_y\tau_y$.

$\vec{h}(k) = (h_y, h_z)$ wind in y-z plane.

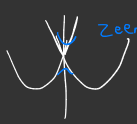
for Kitaev chain. $N = [\mathrm{sgn}(h_z(0)) - \mathrm{sgn}(h_z(\pi))]/2$. $h_z = -\mu - t\cos k$.

必须关闭能隙? $|\mu| < t$.



MZM. coupled energy $E_f \propto e^{-L}$.

1D TSC. SOC. s-wave. Zeeman field. induce equivalent spinless p-wave SC.



non-Abelian statistics

$U(1)$ phase many-body system. degenerate ground state. $|\phi_f\rangle = \underset{\text{matrix in degenerate space}}{U_{fi}} |\phi_i\rangle$. do not commute. non-Abelian

braiding. adiabatic. $H \longrightarrow H_0$.

$$|\psi_f(t)\rangle = \underbrace{e^{-\frac{i}{\hbar}\int dt\, E(t)}}_{\text{dynamical phase}} \overbrace{\vec{B}_0(t)}^{\text{Berry matrix}} |\alpha(t)\rangle.$$

$2N$. seperate far MZM.

$$f_i = \frac{\gamma_{2i-1} + i\gamma_{2i}}{2}. \qquad \varepsilon_i = 0.$$

$$|\phi_i\rangle = |n_1 \cdots n_N\rangle.$$

$$i\hbar \frac{d\psi}{dt} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi . \qquad \frac{d\psi}{dt} = \left(-i\hbar c\vec{\alpha}\nabla + \beta mc^2\right)\psi .$$

Solution: $(i\hbar\tilde{\gamma}^{\mu}\partial_{\mu} - mc)\Psi = 0$.

$c_i^{\dagger}$. generate an electron. (annihlate a hole)

$$c_i^2 = (c_i^{\dagger})^2 = 0. \qquad \{c_i^{\dagger}, c_i\} = 1.$$

$$\{c_i^{\dagger}, c_j\} = c_i^{\dagger}c_j + c_jc_i^{\dagger} = \{c_i, c_j\} = \{c_i^{\dagger}, c_j^{\dagger}\} = 0$$

$$\gamma_{i,1}^2 = \gamma_{i,2}^2 = 1.$$

$$c_i = \frac{1}{2}(\gamma_{i,1} + i\gamma_{i,2}) \qquad \gamma_{i,1} = c_i^{\dagger} + c_i . \qquad \gamma_{i,1}\gamma_{i,2} + \gamma_{i,2}\gamma_{i,1} = 2$$

$$c_i^{\dagger} = \frac{1}{2}(\gamma_{i,1} - i\gamma_{i,2}). \qquad \gamma_{i,2} = i(c_i^{\dagger} - c_i). \qquad \gamma_{i,1}\gamma_{j,2} + \gamma_{j,2}\gamma_{i,1} = 0.$$

$$H = -\mu\sum_i^N c_i^{\dagger}c_i - \sum_i^{N-1}\left(tc_i^{\dagger}c_{i+1} + \Delta c_ic_{i+1} + h.c.\right)$$

$$c_i^{\dagger}c_i = \frac{1}{4}\left(\gamma_{i,1}^2 - i\gamma_{i,2}\gamma_{i,1} + i\gamma_{i,1}\gamma_{i,2} + \gamma_{i,2}^2\right).$$

$$c_i^{\dagger}c_{i+1} =$$

$$\tilde{c}_i = \frac{\gamma_{i+1,1} + i\gamma_{i,2}}{2}.$$

$$\Rightarrow H = -it\sum_{i=1}^{N-1}\gamma_{i,2}\gamma_{i+1,1} = 2t\sum_{i=1}^{N-1}\tilde{c}_i^{\dagger}\tilde{c}_i .$$

0407088

type-II quantum computing.

utilize coherent charge transfer in an incoherent device

i.f states are eigenstates of H. intermediate states formed by coherent superposition

enough coherence to effect SWAP gate. not enough coherence to maintain any superpositions after the initial transfer complete

coherent population transfer — non-adiabatic

control of tunneling to realize π or similar pulses

SNR. pulse switching time. X.

(adiabatic.

quasi-static population transfer

adiabatic passage

$\Downarrow$

simultaneous modulation of at least two system parameters to realize a desired trajectory through the Hilbert space

EM mediated adiabatic passage in quantum computing?

generating entanglement via adiabatic passage?

maintaining constant energy. prevent dynamical phase when QI transported.

donors. $|1\rangle, |2\rangle, |3\rangle$. (position eigenstates) shift gates. barrier gates.

usually gaussian pulses?

$$c = \gamma(x) + \gamma^\dagger(x)$$

$$c^\dagger = -i\left(\gamma(x) - \gamma^\dagger(x)\right)$$

$$ic^\dagger = \gamma(x) - \gamma^\dagger(x)$$

张量积 $\otimes$ 意味着系统的某种局域性？

简并度？

1D TFIM $\quad H = -J\sum_{i=1}^{L} \sigma_i^x \sigma_{i+1}^x - h\sum_{i=1}^{L} \sigma_i^z$.

$S_j^+ = c_j^+ \underbrace{e^{i\pi \sum_{k<j} c_k^+ c_k}}$ JW string

$S_j^- = \underbrace{e^{-i\pi \sum_{k<j} c_k^+ c_k}} c_j$

$S_j^z = c_j^+ c_j - \frac{1}{2}$.

$[S_j^+, S_k^-]$ for $j=k$, $j<k$.

$$[S_j^+, S_k^-] = c_j^+ e^{i\pi \sum_{l<j} c_l^+ c_l} e^{-i\pi \sum_{m<k} c_m^+ c_m} c_k - e^{-i\pi \sum_{m<k} c_m^+ c_m} c_k c_j^+ e^{i\pi \sum_{l<j} c_l^+ c_l}.$$

① $j=k$.

$$[S_j^+, S_k^-] = c_j^+ e^{i\pi \sum_{l<j} c_l^+ c_l} e^{-i\pi \sum_{l<j} c_l^+ c_l} c_j - e^{-i\pi \sum_{l<j} c_l^+ c_l} c_j c_j^+ e^{i\pi \sum_{l<j} c_l^+ c_l}$$

$e^{i\pi (c_1^+ c_1 + c_2^+ c_2 + \cdots c_j^+ c_j)}$.

$c_l^+ c_l + c_l c_l^+ = 1$.

$c_j^+ c_j + c_j c_j^+ = 1$

$$= c_j^+ c_j - c_j c_j^+ = 2n_j - 1$$

n_j occupation number on jth site

$n_j - \frac{1}{2} = S_j^z$? ✓

$-\frac{1}{2}$ or $\frac{1}{2}$. $\longrightarrow$ 0 or 1.

② $j<k$.

$$[S_j^+, S_k^-] = c_j^+ e^{i\pi \sum_{l<j} c_l^+ c_l} e^{-i\pi \sum_{m<k} c_m^+ c_m} c_k - e^{-i\pi \sum_{m<k} c_m^+ c_m} c_k c_j^+ e^{i\pi \sum_{l<j} c_l^+ c_l}.$$

?

$$= c_j c_k^+ e^{-i\pi \sum_{l<j} c_l^+ c_l} e^{-i\pi \sum_{m<k} c_m^+ c_m} - c_k c_j^+ e^{-i\pi \sum_{m<k} c_m^+ c_m} e^{i\pi \sum_{l<j} c_l^+ c_l}$$

$$= \{c_j, c_k^+\} \cdots = 0.$$

$$H = -\mu \sum_i c_i^\dagger c_i - \frac{1}{2}\sum_i \left(t c_i^\dagger c_{i+1} + \Delta c_i c_{i+1} + h.c.\right)$$

$$c_j = \frac{1}{\sqrt{N}} \sum_k e^{ikj} c_k . \quad c_j^\dagger = \frac{1}{\sqrt{N}} \sum_k e^{-ikj} c_k^\dagger .$$

$$\Rightarrow \quad H = -\mu \sum_k c_k^\dagger c_k - \sum_k \left(t\cos k\, c_k^\dagger c_k - \Delta c_k^\dagger c_k^\dagger e^{ik} + c_k c_k e^{-ik}\right)$$

$$H = -\mu \sum_i \left(c_i^\dagger c_i - \frac{1}{2}\right) - \sum_i \left(t c_i^\dagger c_{i+1} + \Delta c_i c_{i+1} + h.c.\right).$$

$$\begin{cases} c_i = \frac{1}{\sqrt{L}} \sum_k e^{ikx_i} c_k \\ c_i^\dagger = \frac{1}{\sqrt{L}} \sum_k e^{-ikx_i} c_k^\dagger \end{cases}$$

periodic condition

$$\Rightarrow \quad H = -\mu \sum_{i,k} \frac{1}{L} c_k^\dagger c_k - \sum_{i,k} \Big(\underbrace{\frac{t}{L} e^{ik(x_{i+1}-x_i)}}_{t_k} c_k^\dagger c_k + \underbrace{\frac{\Delta}{L} e^{ik(x_{i+1}+x_i)}}_{\Delta_k} c_k c_k + h.c. \Big)$$

$$+ \frac{1}{2}\mu L$$

$$= -\mu \sum_k \left(c_k^\dagger c_k - \frac{1}{2}\right) - \sum_k \left(t_k c_k^\dagger c_k + \Delta_k c_k c_k + h.c.\right)$$

$$t\left(c_1^\dagger c_2 + c_2^\dagger c_3 + \cdots c_{L-1}^\dagger c_L + c_L^\dagger c_1\right)$$

$$\frac{1}{\sqrt{L}} \sum_k e^{-ik} c_k^\dagger \cdot \frac{1}{\sqrt{L}} \sum_k e^{2ik} c_k + \cdots$$

$$\frac{1}{L} \sum_k e^{ik} c_k^\dagger c_k + \cdots$$

Tang - Thesis

how quantum information theory can be used to study physical systems

at high / low energies.

low-energy subspaces of quantum many-body systems

error-correcting properties that such subspaces can exhibit

Topological phases are special phases of matter which exhibit long-range

entanglement structure

requirement: ground state space of a topologically ordered model

cannot be distinguished by local operations

⇓ the ground state → low energy subspaces ?

constitutes a quantum error-detecting code against local errors

Note of Kitaev

A 2D quantum system with anyonic excitations can be considered as a quantum computer. Unitary operations can be performed by moving the exitations around each other. Measurements can be performed by joining excitations in pairs and observing the result of fusion.

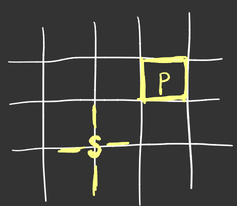
Magnetism : spin flips. (similar to error correction for the repetition code.)

$\Downarrow$

classical physical level $\to$ quantum case ?

need a quantum code with local stabilizer operators

1 Toric codes and the corresponding Hamiltonians



$k \times k$ square lattice on the torus

A general pure state on $2 \otimes 2$ space is

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle. \qquad |\alpha|^2+|\beta|^2+|\gamma|^2+|\delta|^2=1.$$

If apply U to the basis of A space

$$U|0\rangle = |e_0\rangle. \qquad U|1\rangle = |e_1\rangle. \qquad |e_0\rangle\langle 0|$$

$$|\psi\rangle = |e_0\rangle|\tilde{\varphi}_0\rangle + |e_1\rangle|\tilde{\varphi}_1\rangle.$$

$$\rho^A = |e_0\rangle\langle e_0|\langle\tilde{\varphi}_0|\tilde{\varphi}_0\rangle + |e_0\rangle\langle e_1|\langle\tilde{\varphi}_1|\tilde{\varphi}_0\rangle + |e_1\rangle\langle e_0|\langle\tilde{\varphi}_0|\tilde{\varphi}_1\rangle + |e_1\rangle\langle e_1|\langle\tilde{\varphi}_1|\tilde{\varphi}_1\rangle$$

Also $\rho^A = \sigma_0|e_0\rangle\langle e_0| + \sigma_1|e_1\rangle\langle e_1|.$

$$|\psi\rangle = \sqrt{\sigma_0}\,|e_0\rangle\frac{1}{\sqrt{\sigma_0}}|\varphi_0\rangle + \sqrt{\sigma_1}\,|e_1\rangle\frac{1}{\sqrt{\sigma_1}}|\varphi_1\rangle$$

$$U|\psi\rangle = \alpha|e_0\rangle|0\rangle + \beta|e_0\rangle|1\rangle + \gamma|e_1\rangle|0\rangle + \delta|e_1\rangle|1\rangle$$

$$= |e_0\rangle(\alpha|0\rangle+\beta|1\rangle) + |e_1\rangle(\gamma|0\rangle+\delta|1\rangle)$$

$$\rho_A = \sigma_0|e_0\rangle\langle e_0| + \sigma_1|e_1\rangle\langle e_1|. \qquad \text{表象变换?}$$

$$\left(|e_0\rangle\langle e_0| + |e_1\rangle\langle e_1|\right)$$

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle.$$

$$= \alpha|e_0\rangle\langle e_0|0\rangle|0\rangle + \alpha|e_1\rangle\langle e_1|0\rangle|0\rangle + \cdots$$

$$= \alpha\langle e_0|0\rangle|e_0\rangle|0\rangle + \cdots$$

$\rho_A = \sigma_0 |e_0\rangle\langle e_0| + \sigma_1 |e_1\rangle\langle e_1|$

变换 $\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)$

$|0\rangle\langle 0|e_0\rangle\langle e_0|0\rangle\langle 0|$

$\langle 0|e_0\rangle\, \sigma_0 \langle e_0|0\rangle\, |0\rangle\langle 0| +$

设 $U = \sum_i |i\rangle\langle e_i|$ $\quad V = \sum_j |j\rangle\langle e_j|$ $\quad |\alpha\rangle$ 与 $|e_i\rangle$ $|e_j\rangle$ 在一个space.

$$UDV^\dagger = \sum_i |i\rangle\langle e_i| \sum_\alpha \lambda_\alpha |\alpha\rangle\langle \alpha| \sum_j |e_j\rangle\langle j|$$

$$= \sum_i |i\rangle \lambda_i \langle e_i| \sum_j |e_j\rangle\langle j| = \sum_i \lambda_i |i\rangle\langle j|.$$

$U = \sum_i |e_i\rangle\langle i|.$ $\quad V = \sum_j |e_j\rangle\langle j|.$

$$UDV^\dagger = \sum_i |e_i\rangle\langle i| \sum_\alpha \lambda_\alpha |\alpha\rangle\langle \alpha| \sum_j |j\rangle\langle e_j|$$

$$= \sum_i |e_i\rangle \lambda_i \langle i| \sum_j |j\rangle\langle e_j| = \sum_i \lambda_i |e_i\rangle\langle e_i|.$$

$$UD = \sum_i |e_i\rangle \lambda_i \langle i| = \sum_i \lambda_i |e_i\rangle\langle i|$$

$$\rho = \alpha|00\rangle + \delta|01\rangle + \gamma|10\rangle + \beta|11\rangle . \qquad \alpha = \frac{1}{\sqrt{2}} \quad \delta = \frac{1}{\sqrt{3}} \quad \beta = \frac{1}{\sqrt{6}}$$

$$\rho_A = \mathrm{Tr}_B(\rho) =$$

$$p_0|e_0\rangle\langle e_0| + p_1|e_1\rangle\langle e_1|$$

$$\rho_A = \mathrm{Tr}_B\left(|\psi\rangle_{AB}\langle\psi|_{AB}\right)$$

$$U = |0\rangle\langle e_0| + |1\rangle\langle e_1|$$

$$= \begin{pmatrix} |\alpha|^2 + |\delta|^2 & \alpha\gamma^* + \beta^*\delta \\ \alpha^*\gamma + \beta\delta^* & |\gamma|^2 + |\beta|^2 \end{pmatrix}$$

$$p_0|0\rangle\langle e_0| + p_1|1\rangle\langle e_1|$$

$$V = |e_0\rangle\langle 0| + |e_1\rangle\langle 1|$$

$$p_0|0\rangle\langle 0|$$

$$\frac{5}{6} \qquad \frac{1}{3\sqrt{2}}$$

$$\frac{1}{3\sqrt{2}} \qquad \frac{1}{6}$$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ 对角化?

$\|$

UDV

举例 $$\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{10} & \\ & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$$

$$|e_0\rangle = U^\dagger|0\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \sqrt{5} & 0 \\ \sqrt{5} & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} .$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sqrt{10}|e_0\rangle\langle e_0| = \sqrt{10} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = \sqrt{10}\Big(\frac{1}{2}$$

$$U = \sum_i |i\rangle\langle e_i|$$

$$|0\rangle\langle e_0| + |1\rangle\langle e_1| \ ?$$

$$U = |0\rangle\langle e_0| + |1\rangle\langle e_1|$$

$$U|e_0\rangle = |0\rangle \qquad U|e_1\rangle = |1\rangle$$

$$u_{00} = \langle e_0|0\rangle \qquad u_{01} = \langle e_0|1\rangle$$

$$u_{10} = \langle e_1|0\rangle \qquad u_{11} = \langle e_1|1\rangle$$

$$u_{ij} = \langle \alpha_i|\beta_j\rangle$$

$$\langle e_i|j\rangle$$

$$\begin{pmatrix}\langle e_0|\\ \langle e_1|\end{pmatrix}\begin{pmatrix}|0\rangle & |1\rangle\end{pmatrix}$$

$$U=\sum_i|\beta_i\rangle\langle\alpha_i|$$

$$|i\rangle\langle e_i|$$

$$\begin{pmatrix}1&2\\1&2\end{pmatrix}=\begin{pmatrix}\frac{1}{\sqrt2}&-\frac{1}{\sqrt2}\\ \frac{1}{\sqrt2}&\frac{1}{\sqrt2}\end{pmatrix}\begin{pmatrix}\sqrt{10}&\\&0\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt5}&\frac{2}{\sqrt5}\\ -\frac{2}{\sqrt5}&\frac{1}{\sqrt5}\end{pmatrix}$$

$U|e_0\rangle=|0\rangle$. 矩阵? $|e\rangle$表象?

$$\begin{pmatrix}\langle e_0|0\rangle & \langle e_0|1\rangle\\ \langle e_1|0\rangle & \langle e_1|1\rangle\end{pmatrix}\begin{pmatrix}|\phi\rangle\end{pmatrix}=\begin{pmatrix}\langle e_0|0\rangle\\ \langle e_1|0\rangle\end{pmatrix}$$

$$\sqrt{10}\,|e_0\rangle\langle e_0|+0\,|e_1\rangle\langle e_1|$$

$$\langle e_0|0\rangle|e_0\rangle+\langle e_1|0\rangle|e_1\rangle=\mathbb{1}|0\rangle$$

与 $|0\rangle,|1\rangle$ 表象中的 $\begin{pmatrix}1&2\\1&2\end{pmatrix}$ 之间的变换.

那么 $|0\rangle,|1\rangle$ 与 $|e_0\rangle,|e_1\rangle$ 的关系?

$$U=\begin{matrix}\langle e_0|0\rangle|e_0\rangle\langle e_0|+\langle e_0|1\rangle|e_0\rangle\langle e_1|\\ +\langle e_1|0\rangle|e_1\rangle\langle e_0|+\langle e_1|1\rangle|e_1\rangle\langle e_1|\end{matrix}\quad ?$$

$$\left(\frac{1}{\sqrt2}|e_0\rangle\langle e_0|-\frac{1}{\sqrt2}|e_0\rangle\langle e_1|+\frac{1}{\sqrt2}|e_1\rangle\langle e_0|+\frac{1}{\sqrt2}|e_1\rangle\langle e_1|\right)\qquad \sqrt{10}|e_0\rangle\langle e_0|$$

$$\left(\sqrt5|e_0\rangle\langle e_0|+\sqrt5|e_1\rangle\langle e_1|\right)$$

$$u_{ij}=\langle\alpha_i|\beta_j\rangle.\qquad \sum_{ij}\langle\alpha_i|\beta_j\rangle|\alpha_i\rangle\langle\alpha_j|$$

$$\left(\frac{1}{\sqrt5}|e_0\rangle\langle e_0|+\frac{2}{\sqrt5}|e_0\rangle\langle e_1|+\cdots\right)$$

$$=\ |e_0\rangle\langle e_0|\cdots$$

都是在 $|e_0\rangle.|e_1\rangle$ 表象.

在 $|0\rangle,|1\rangle$ 中?

基矢量的变换?

$$V\begin{pmatrix}1&2\\1&2\end{pmatrix}U^\dagger=\begin{pmatrix}&\\&\end{pmatrix}$$

$$\sqrt{10}|e_0\rangle\langle e_0|+0|e_1\rangle\langle e_1|.\qquad |e_0\rangle=a|0\rangle+b|1\rangle.$$

$$\begin{pmatrix}a\\b\end{pmatrix}\begin{pmatrix}a^*&b^*\end{pmatrix}$$

$$\sqrt{10}\begin{pmatrix}|a|^2&ab^*\\a^*b&|b|^2\end{pmatrix}=\begin{pmatrix}1&2\\1&2\end{pmatrix}$$

$$\sqrt{14+6\sqrt5}$$

不成立!

$$(3+\sqrt5)^2=14+6\sqrt5$$

$X^B = U^\dagger X^A U.$

$X^A = x_{00} |e_0\rangle\langle e_0| + x_{01} |e_0\rangle\langle e_1|$
$+ x_{10} |e_1\rangle\langle e_0| + x_{11} |e_1\rangle\langle e_1|.$

$U, U^\dagger$ also in $|e_0\rangle, |e_1\rangle$ representation

How do we get $|0\rangle, |1\rangle$? (In B representation)

$|\beta_j\rangle = U |\alpha_j\rangle.$ $\quad u_{ij} = \langle\alpha_i|\beta_j\rangle.$ $\quad U = \sum_{ij} \langle\alpha_i|\beta_j\rangle |\alpha_i\rangle\langle\alpha_j|$?

$U = \sum_j |\beta_j\rangle\langle\alpha_j|.$? ?

混淆左右矢表示与矩阵表示

基矢量的变换? $U|e_0\rangle = |0\rangle$

$$\begin{pmatrix} 4+2\sqrt{5} & 4-2\sqrt{5} \\ 3+\sqrt{5} & 3-\sqrt{5} \end{pmatrix}$$

$\dfrac{1+\sqrt{5}}{\sqrt{10+2\sqrt{5}}}$ $\quad \dfrac{1-\sqrt{5}}{\sqrt{10-2\sqrt{5}}}$

$\dfrac{2}{\sqrt{5+\sqrt{5}}}$ $\quad \dfrac{2}{\sqrt{5-\sqrt{5}}}$