置换对称性

粒子不可区分

$|k'\rangle|k''\rangle$ $|k''\rangle|k'\rangle$ 而 k'、k'' 是可区分的右矢

所有具有 $c_1|k'\rangle|k''\rangle + c_2|k''\rangle|k'\rangle$ 形式的右矢

将导致完全相同的本征值集合 $\Longrightarrow$ 交换简并

困难：一个可观测量完备集本征值的规定不能完全确定这个态右矢

$$A_1|a'\rangle|a''\rangle = a'|a'\rangle|a''\rangle \qquad P_{12}A_1P_{12}^{-1}\underbrace{P_{12}|a'\rangle|a''\rangle}_{|a''\rangle|a'\rangle} = a'\underbrace{P_{12}|a'\rangle|a''\rangle}_{|a''\rangle|a'\rangle}$$

$$A_2|a'\rangle|a''\rangle = a''|a'\rangle|a''\rangle$$

仅当 $P_{12}A_1P_{12}^{-1} = A_2$ 时一致　P_{12} 一定会交换可观测量的粒子标号

P_{12} 与 H 对易，运动常数。开始对称（反对称）则保持不变

二次量子化

多粒子态矢量 $|n_1, n_2, \cdots n_i, \cdots\rangle$ Fock space

(为何这样选态矢?)

隐含假设. 存在一组无相互作用的基.

$|0,0,\cdots 0\cdots\rangle = |\vec{0}\rangle$. 真空.

$|0,0,\cdots n_i=1,\cdots\rangle \equiv |k_i\rangle$. 单粒子态.

定义场算符 $a_i^\dagger$.

$$a_i^\dagger |n_1, n_2, \cdots n_i, \cdots\rangle \propto |n_1, n_2, \cdots n_i+1, \cdots\rangle$$

$$a_i^\dagger |\vec{0}\rangle = |k_i\rangle. \qquad 1 = \langle k_i | k_i\rangle = \langle \vec{0} | a_i a_i^\dagger |\vec{0}\rangle = \langle 0 | \underbrace{a_i | k_i\rangle}_{\text{湮灭}} = 0$$

$$a_i |k_j\rangle = \delta_{ij} |\vec{0}\rangle.$$

引入交换对称性. $|k_i\rangle$ $|k_j\rangle$

对两粒子态 $a_i^\dagger a_j^\dagger |\vec{0}\rangle = \pm a_j^\dagger a_i^\dagger |\vec{0}\rangle$.

$$a_i^\dagger a_j^\dagger - a_j^\dagger a_i^\dagger = [a_i^\dagger, a_j^\dagger] = 0. \quad \text{Boson} \quad \xrightarrow{\text{取共轭}} \quad [a_i, a_j] = 0$$

$$\underbrace{a_i^\dagger a_j^\dagger + a_j^\dagger a_i^\dagger}_{} = \{a_i^\dagger, a_j^\dagger\} = 0. \quad \text{Fermion} \quad \{a_i^\dagger, a_j^\dagger\} = 0$$

$\searrow$ $a_i^\dagger a_i^\dagger = 0$. 包含 Pauli 不相容原理

a_i 与 $a_j^\dagger$ 的对易关系? 定义 $N_i = a_i^\dagger a_i$. 对处于单粒子态 $|k_i\rangle$ 粒子数计数.

Boson	Fermion
$a_i^\dagger a_j^\dagger - a_j^\dagger a_i^\dagger = [a_i^\dagger, a_j^\dagger] = 0$	$a_i^\dagger a_j^\dagger + a_j^\dagger a_i^\dagger = \{a_i^\dagger, a_j^\dagger\} = 0$
$a_i a_j - a_j a_i = [a_i, a_j] = 0$	$a_i a_j + a_j a_i = \{a_i, a_j\} = 0$
$a_i a_j^\dagger - a_j^\dagger a_i = [a_i, a_j^\dagger] = \delta_{ij}$	$a_i a_j^\dagger + a_j^\dagger a_i = \{a_i, a_j^\dagger\} = 0.$

多费米系统

互换产生π相移

由 Pauli 不相容原理 和 只存在跃迁项的 H 刻画.

可由反对易算符描述

△ 晶格上的无自旋费米系统

H 基矢 $\{|n_{i_1}, n_{i_2}, \cdots\rangle\}$. $n_i = 0,1$, 格点 i 的费米子数

为每个格点引入

$$\sigma_i^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \frac{1}{2}(\sigma_x - i\sigma_y).$$ 湮灭 玻色子算符?

$$\sigma_i^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \frac{1}{2}(\sigma_x + i\sigma_y).$$ 生成

Fermi 系统的 Hamiltonian: 实际上是硬核 Boson 系统 或 spin-1/2 系统

$$H = \sum_{\langle ij\rangle} (t_{ij}\sigma_i^+\sigma_j^- + h.c.).$$

△ 满足 Pauli 不相容原理的 Fermi Hilbert 空间

△ 特别的 Hamiltonian

$$H_f = \sum_{\langle ij\rangle} \left[t_{ij}(\{\sigma_i^z\})\sigma_i^+\sigma_j^- + h.c.\right].$$

简化. 排序格点: $(i_1, i_2, \cdots i_a, \cdots)$

$$C_{i_a} = \sigma_{i_a}^- \prod_{b<a} \sigma_{i_b}^z.$$

$$\{C_i, C_j\} = \{C_i^+, C_j^+\} = 0$$

$$\{C_i, C_j^+\} = \delta_{ij}.$$

$$H_f = \sum_{\langle ij\rangle} (t_{ij} C_i^+ C_j + h.c.).$$

$\sigma^{x,y,z} \longrightarrow c_i$. Jordan-Wigner Transformation

Fermion 是非局域激发.

c_{2j-1}, c_{2j} different site

phase error $a_j^{\dagger} a_j = \frac{1}{2}(1 + i c_{2j-1} c_{2j})$

$$H = H_0 + H'.$$

$$H_0 = \sum_{i=1}^{N-1} K(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+)$$

$g \ll K$ $\quad$ H' perturbation

$$H' = g(S_0^+ S_1^- + S_{N+1}^+ S_N^- + h.c.)$$

introduce fermi operator

$$C_i = e^{i\pi \sum_{0}^{i-1} S_j^+ S_j^-} S_i^-.$$

$$H_0 \Rightarrow \sum_{i=1}^{N-1} K(C_i^+ C_{i+1} + C_i C_{i+1}^+).$$

conservation of S_z $\rightarrow$ conservation of fermion number

orthogonal transformation to diagonalize

$$f_k^+ = \frac{1}{A} \sum_{j=1}^{N} \sin\frac{jk\pi}{N+1} C_j^+.$$

$$A = \sqrt{\frac{N+1}{2}}.$$

$$\Rightarrow H_0 = \sum_{k=1}^{N} E_k f_k^+ f_k. \qquad E_k = 2K\cos\frac{k\pi}{N+1}.$$

$$\Rightarrow H' = \sum_{k=1}^{N} t_k \left(C_0^+ f_k + (-1)^{k-1} C_{N+1}^+ f_k + h.c.\right) \qquad t_k = \frac{g}{A}\sin\frac{k\pi}{N+1}.$$

odd N. exists a single zero energy fermionic mode $k = z \equiv \frac{N+1}{2}$.

two end spins resonantly coupled to zero energy fermion by H'.

assumption $t_z \sim g/A \ll$ fermion detuning $|E_z - E_{z\pm1}| \sim K/N$.

other modes can be neglected.

Upon absorbing phase factor $(-1)^{z-1}$ into C_{N+1}.

$$H_{eff} = t_z(C_0^+ f_z + C_{N+1}^+ f_z + h.c.)$$

resonant fermionic tunneling

Unitary evolution under $\tau = \frac{\pi}{\sqrt{2} t_z}$.

$$U_{eff} = e^{-i\tau H_{eff}} = (-1)^{f_z^+ f_z} \left(1 - (c_0^+ + c_{N+1}^+)\right)(c_0 + c_{N+1}).$$

Upon projection to subspace $\{(1, c_0^+, c_{N+1}^+, c_0^+ c_{N+1}^+)|00\rangle_{0,N+1}\}$

the effective resolution

$$U_{eff}^{fermi} = (-1)^{n_0 + n_{N+1} + n_z} (-1)^{n_0 n_{N+1}} \, SWAP_{0,N+1}$$

entangled through a CPHASE gate. $CP_{0,N+1} = (-1)^{n_0 n_{N+1}}$.

$$S_i^+ S_j^- + S_i^- S_j^+$$

$$(S_{ix} + iS_{iy}) \cdot (S_{jx} - iS_{jy}) + (S_{ix} - iS_{iy}) \cdot (S_{jx} + iS_{jy})$$

$$(S_{ix} S_{jx} + S_{iy} S_{jy}) \longrightarrow \vec{S}_i \cdot \vec{S}_j$$

$\underline{c_i^+ c_{i+1} + c_i c_{i+1}^+}$? derive

$$H_G = -\sum_{\langle i,j\rangle} J_{ij}\, \vec{\sigma}_i \cdot \vec{\sigma}_j - \sum_{i=1}^{N} B_i \sigma_z^i . \qquad [H_G, \sigma_z] = 0$$

arbitrary FM with isotropic Heisenberg interactions

Initialize to ground state $|\vec{0}\rangle = |00\cdots0\rangle$

easy for FM system by cooling. set $E_0 = 0$.

$|\vec{j}\rangle = |0\cdots010\cdots0\rangle$.

channel spins

$\vec{j} = 1, 2, \cdots s, \ldots r, \cdots N$.

Alice

sender

Bob

receiver

$t=0$. sth spin. $|\psi_{in}\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\theta}\sin\frac{\theta}{2}|1\rangle$.

State of whole chain $|\psi(0)\rangle = \cos\frac{\theta}{2}|\vec{0}\rangle + e^{i\phi}\sin\frac{\theta}{2}|\vec{s}\rangle$.

Bob wait for $|\psi(0)\rangle$ to involve to a final state

as close as possible to $\cos\frac{\theta}{2}|\vec{0}\rangle + e^{i\phi}\sin\frac{\theta}{2}|\vec{r}\rangle$.

$$|\psi(t)\rangle = \cos\frac{\theta}{2}|\vec{0}\rangle + e^{i\varphi}\sin\frac{\theta}{2}\sum_{j=1}^{N}\langle j|e^{-iH_G t}|s\rangle|j\rangle$$

?

output state $\mathrm{Tr}_{1,2\cdots N-1}\left(|\psi(t)\rangle\langle\psi(t)|\right)$

$$\rho_{out} = P(t)\,|\psi_{out}(t)\rangle\langle\psi_{out}(t)| + (1-P(t))\,|0\rangle\langle 0|$$

$e^{-iHt}|\psi\rangle$ for t-independent H_G. ✓

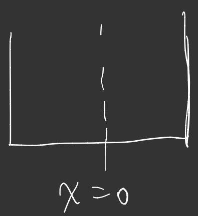
$$e^{-iHt}|\vec{r}\rangle = ?$$

懂了. $|\vec{s}\rangle$ 只变成 $|\vec{j}\rangle$ 意思是 spin 数守恒.--

$$\underline{\sum_{j=1}^{N}|j\rangle\langle j|\,e^{-iH_G t}}\,|s\rangle \quad ✓$$

Assume $J_{ij} = J/2\ \delta_{i+1,j}$. $\quad B_i = B$.

eigenstates $|\tilde{m}\rangle_L = a_m \sum_{j=1}^{N}\cos\left\{\frac{\pi}{2N}(m-1)(2j-1)\right\}|\vec{j}\rangle$.



$$\phi_k(-x) = (-1)^k \phi_k(x).$$

alternating parity

$$\psi(x,t) = \sum_k c_k e^{-iE_k t} \phi_k(x).$$

$E_k \sim k^2$. commensurate energy

If we choose $t = \tau$. $\tau = \frac{k^2 \pi}{E_k}$.

mirror inverts

$$\psi(x,\tau) = \sum_k c_k (-1)^k \phi_k(x) = \sum_k c_k \phi_k(-x) = \psi(-x).$$

analogy [position of particle in a line
single spin flip in a background of aligned spins

x $\psi(x)$

$j = 1, \cdots N$ $\sum_j c_j |j\rangle$.

If $J_{j,j+1} = J_{N-j,N-j+1}$.

Fermi liquid interaction

LAS VEGAS LINQ

e-p coupling

stacked flat bands

graphene superlattice potential

topological state under decoherence

2D cluster state SPT

which aspect of teleportation ?

$$\gamma = \sum u_i c_i + v_i c_i^\dagger$$

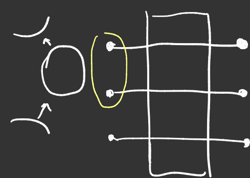
u_i, v_i

same state at different location ---

eject a fermion

STM

not --- happen

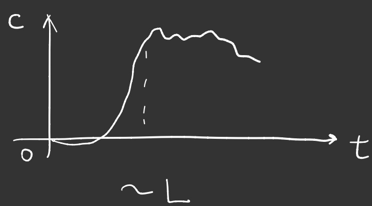


correlation function

n_1 n_2

$$\langle n_2(t)\, n_1(0)\rangle \to \langle n_2(t)\rangle\langle n_1(0)\rangle$$

XX+YY



$S_x S_x$ $h S_z$

Ising

$$t c_i c_{i+1}^\dagger + \Delta c_i c_{i+1}^\dagger \qquad \sqrt{t^2\cos^2 k + \Delta^2 \sin^2 k} \qquad E \to 0$$

epr-like setup using majorana wire

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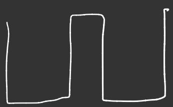
$|\psi\rangle$

state described by wavefunction well-seperated

↳ eigenenergy seperated

V

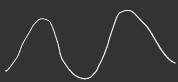


create a particle in ground state ?

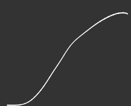
~~populate groud state~~

→ place the particle into a superposition of symmetric & anti-symmetric states

→ tunnel to symmetrize its state



groundstate



antisymmetric state ?

(close to degenerate)

"isolated" Single-particle states ?

— Dirac equation (interacting with topologically non-trivial backgrounds)

1D. $\left[i\gamma^\mu \partial_\mu + \phi(x)\right]\psi(x,t) = 0.$

$$\phi(x) = \begin{cases} \phi_0 & x<0,\ x>L \\ -\phi_0 & 0<x<L. \end{cases}$$

$$i\begin{pmatrix} & \frac{d}{dx}+\phi(x) \\ \frac{d}{dx}-\phi(x) & \end{pmatrix}\begin{pmatrix} U_E(x) \\ V_E(x)\end{pmatrix} = E\begin{pmatrix} U_E(x) \\ V_E(x)\end{pmatrix}$$

two bound states

$$E_+ \approx \phi_0 e^{-\phi_0 L}. \quad \psi_+ \approx \sqrt{\phi_0}\begin{pmatrix} e^{-\phi_0|x|} \\ -ie^{-\phi_0|L-x|}\end{pmatrix} + O(e^{-\phi_0 L})$$

$$E_- \approx -\phi_0 e^{-\phi_0 L}. \quad \psi_- \approx \sqrt{\phi_0}\begin{pmatrix} e^{-\phi_0|x|} \\ ie^{-\phi_0|L-x|}\end{pmatrix} + O(e^{-\phi_0 L})$$

many fermion system. E_- filled. E_+ empty.

fermion / anti-fermion excited by

create a fermion by populating the positive energy bound state. has ψ_+. 

anti-fermion ψ_- > degenerate ...?

$\phi_0 L$ large. $E_+, E_- \to 0$.

$$\psi_0 = \frac{1}{\sqrt{2}}\left(e^{iE_0 t}\psi_+ + e^{-iE_0 t}\psi_-\right) \approx \sqrt{2\phi_0}\begin{pmatrix} e^{-\phi_0|x|}\cos E_0 t \\ e^{-\phi_0|L-x|}\sin E_0 t \end{pmatrix} + O(e^{-\phi_0 L})$$

for $t \ll \frac{1}{E_0}$. around $x=0$.

$$\psi_L = \frac{1}{\sqrt{2}i}\left(e^{iE_0 t}\psi_+ - e^{-iE_0 t}\psi_-\right) \approx \sqrt{2\phi_0}\begin{pmatrix} e^{-\phi_0|x|}\sin E_0 t \\ -e^{-\phi_0|L-x|}\cos E_0 t \end{pmatrix} + O(e^{-\phi_0 L})$$

near $x=L$.

relevant state could be anything but the ground state that has

$\psi_-(x)$ populated and $\psi_+(x)$ empty

dump fermion

$x=0$

$\Rightarrow$ populate ψ_0 rather than ψ_+

degenerate fermion / anti-fermion state

Majorana. better.

Hamiltonian of majorana. map + energy to − energy states

$$\psi_{-E}(x) = \psi_E^*(x).$$

fermion, anti-fermion $\longrightarrow$ majorana fermion number not conserved (?) parity conserved.

Majorana fermion. only one bound state. wavefunction ψ_+. occupy or empty.

$(-1)^F = -1$. occupy. $(-1)^F = 1$. empty

ψ_0. ψ_L. do not have definite fermion parity.

begin with system with quantum state an eigenstate of fermion parity

Hard? decompose. rapidly remixed by EM interactions.

SC. decomposition more efficiently. charge screened.

Bogoliubov quasi-fermions can be majorana fermions when the

SC condensate is parity-odd. (p-wave e.g.)

mid-gap bound states. — Andreev states (at surface)

p-wave SC. giving vortices non-Abelian fractional statistics

weak coupling to SC

— electrons can enter and leave the wire

as Cooper pairs

$$H = \sum_{n=1}^{L} \left(\frac{t}{2} a_{n+1}^{\dagger} a_n + \frac{t^*}{2} a_n^{\dagger} a_{n+1} + \frac{\Delta}{2} a_{n+1}^{\dagger} a_n^{\dagger} + \frac{\Delta^*}{2} a_n a_{n+1} + \mu a_n^{\dagger} a_n \right)$$

$|\mu| < |t|$. $|\Delta| < |t|$.

$t = |t| e^{i\theta}$. $\quad \Delta = |\Delta| e^{2i\theta}$.

$a_n \longrightarrow e^{i(\phi+\theta)} a_n$ odd n

$a_n \longrightarrow e^{i(\phi-\theta)} a_n$ even n

$$i \frac{d}{dt} a_n = \frac{t}{2}(a_{n+1} + a_{n-1}) - \frac{\Delta}{2}(a_{n+1}^{+} - a_{n-1}^{+}) + \mu a_n .$$

decompose $a_n = b_n + i c_n$. $\quad \psi_n = \begin{pmatrix} b_n \\ c_n \end{pmatrix}$. $\quad \psi_n(t) = e^{i\omega t} \psi_n$.

$$\psi_{n+2} = -N\psi_{n+1} - M\psi_n .$$

$$\begin{pmatrix} 2\mu & \frac{2i\omega}{t-\Delta} \\ \frac{-2i\omega}{t+\Delta} & \frac{2\mu}{t+\Delta} \end{pmatrix} \quad \begin{pmatrix} \frac{t+\Delta}{t-\Delta} & \\ & \frac{t-\Delta}{t+\Delta} \end{pmatrix}$$

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adiabatic transfer protocols

vs

resonant techniques

STIRAP stimulated Raman adiabatic passage

coherent tunneling by adiabatic passage

robust against pulse area & timing errors

useful when interact via lossy intermediate

con: slow, suffer from dissipation / noise in source / target

speed up? counteradiabatic control (transitionless driving)

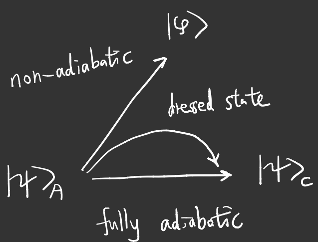
modification of H_0 to compensate for nonadiabatic errors

In principle transitionless driving would allow a perfect state transfer

but: require direct coupling of S/T, or coupling not available in H_0.

constructing dynamical invariants? optimal quantum control?

allow for perfect state transfer even in nonadiabatic regime



develop simple and effective pulses for speeding up adiabatic state transfer in Λ-system

construct $H(t)$. instantaneous (adiabatic) eigenstates

$$\hat{H}(t)|\varphi_k(t)\rangle = E_k(t)|\varphi_k(t)\rangle.$$

$\{|\varphi_{mj}(t)\rangle\}_{j=0}^{n}$ serve as "medium" states.

$$|\varphi_{mj}(t_i)\rangle = |\beta_j\rangle_A \otimes |\chi_i\rangle_{B,C}$$

$$|\varphi_{mj}(t_f)\rangle = |\chi_f\rangle_{A,B} \otimes |\gamma_j\rangle_C$$

if the evolution is perfectly adiabatic ($\tau \gg 1/\Delta E$)

move to adiabatic frame (adiabatic eigenstates are time-independent)

$$\hat{U}(t) = \sum_k |\varphi_k\rangle\langle\varphi_k(t)|.$$

$$\hat{H}_{ad}(t) = \hat{H}_0(t) + \underbrace{\hat{W}(t)} = \sum_k E_k(t)|\varphi_k\rangle\langle\varphi_k| + i\frac{d\hat{U}(t)}{dt}\hat{U}^\dagger(t).$$

generically have off-diagonal matrix elements

magnitude $\uparrow$ as $\tau\downarrow$. $\Rightarrow$ imperfect state transfer

correcting nonadiabatic errors

$\underbrace{\hat{H}_{mod}(t)} = \hat{H}(t) + \underbrace{\hat{H}_c(t)}$. does not involve couplings that cannot be experimentally implemented

no attainably large coupling strength

Δ the corrected dynamics only needs to evolve the system from correct state t_i to t_f

I) a new basis of dressed states $|\tilde{\varphi}_k(t)\rangle$ formally defined by $V(t)$

$$|\tilde{\varphi}_k(t)\rangle = \hat{V}^\dagger(t)|\varphi_k\rangle.$$

II) a control field $\hat{H}_c(t)$ added

Constraints:

Δ $\hat{V}^\dagger(t_f)|\varphi_{m_j}\rangle = \hat{V}^\dagger(t_i)|\varphi_{m_j}\rangle = |\varphi_{m_j}\rangle$.

Δ for all j. evolution of $|\varphi_{m_j}(t)\rangle$ is trivial in the basis defined by $\hat{V}(t)$. $|\tilde{\varphi}_{m_j}(t)\rangle\langle\tilde{\varphi}_{m_j}(t)|$ conserved.

$$\hat{H}_{new}(t) = \hat{V}\hat{H}_{ad}(t)\hat{V}^\dagger + \hat{V}\hat{U}\hat{H}_c(t)\hat{U}^\dagger\hat{V}^\dagger + i\frac{d\hat{V}}{dt}\hat{V}^\dagger.$$

$\langle\tilde{\varphi}_{m_j}|\hat{H}_{new}|\tilde{\varphi}_k\rangle = 0$. for $1\le k\le n$. $k\ne m_j$. (H_c cancel!!)

special case: transitionless driving. $\hat{V}(t) = \mathbb{1}$. $\hat{H}_c = -\hat{U}^\dagger\hat{W}\hat{U}$.

STIRAP. $A \underset{\leftarrow}{\overset{\rightarrow}{=}} B \rightarrow C$

adiabatic eigenstates

two bright states $|\varphi_\pm(t)\rangle$. $E_\pm(t) = \pm\Omega(t)$.

dark state. $|\varphi_D(t)\rangle$. $E_D = 0$.

$$H(t) = \underset{\text{pump}}{\Omega_p(t)\,|B\rangle\langle A|} + \underset{\text{stokes}}{\Omega_s(t)\,|B\rangle\langle C|} + h.c.$$

$$|\varphi_D(t)\rangle = \cos\theta(t)\,|A\rangle + \sin\theta(t)\,|C\rangle. \qquad |\varphi_0(t)\rangle = |000\rangle. \quad E_0(t) = 0.$$

$$U_{ad} = \begin{array}{c} \begin{pmatrix} \sin\theta(t)/\sqrt{2} & -1/\sqrt{2} & -\cos\theta(t)/\sqrt{2} \\ \cos\theta(t) & 0 & \sin\theta(t) \\ \sin\theta(t)/\sqrt{2} & 1/\sqrt{2} & -\cos\theta(t)/\sqrt{2} \end{pmatrix} \end{array} \begin{array}{l} |\varphi_+\rangle \\ |\varphi_D\rangle \\ |\varphi_-\rangle \end{array}$$

$\theta(t_i) = 0$, $\theta(t_f) = \pi/2$.

protocol time reduced. more corrupted.

$$\hat{H}_{ad}(t) = \Omega(t)\hat{M}_z + \underbrace{\dot{\theta}(t)\hat{M}_y}_{\text{non-adabatic couplings}}.$$

$$\hat{M}_z = |\varphi_+\rangle\langle\varphi_+| - |\varphi_-\rangle\langle\varphi_-|.$$

$$\hat{M}_x = (|\varphi_-\rangle - |\varphi_+\rangle)\langle\varphi_D|/\sqrt{2} + h.c.$$

$$[M_p, M_q] = i\varepsilon^{pqr} M_r.$$

$$g_x(t) = \frac{\dot{\mu}}{\cos\xi} - \dot{\theta}\tan\xi.$$

$$g_z(t) = -\Omega + \dot{\xi} + \frac{\dot{\mu}\sin\xi - \dot{\theta}}{\tan\mu\cos\xi}.$$

$$H_{new} = -\frac{\dot{\theta} + \dot{\xi} + \dot{\mu}\sin\xi}{\sin\mu\cos\xi}\hat{M}_z$$

$$= \tilde{E}(t)\hat{M}_z.$$

$$|\psi(t)\rangle = U_{ad}^\dagger V_g^\dagger(t)\exp\left(-i\int_{t_i}^{t} dt'\,\tilde{E}(t')\hat{M}_z\right) V_g(t_i)\,U_{ad}(t_i)\,|\psi(t_i)\rangle.$$

topological SC. nontrivial. distinct from ordinary SC and vacuum.

↳ phase. must exhibit localized modes at boundary.

Dirac matrix. (Majorana representation). real field solution. zero-energy stationary state solution

↳ U(1) gauge symmetry ✗. lepton not conserved. charge neutral. spin 1/2.

Fermi liquid. $\frac{\text{fermi surface} \quad \text{electronic excitation}}{\text{hole-type excitation}}$ → quasi-anti-particle?

⇒ Bogliubov quasi-particle in SC?

s-wave SC. $H^{s}_{pair} = \sum_{k} \Delta_s C_{\vec{k},\uparrow} C_{-\vec{k},\downarrow} + h.c.$

$\xrightarrow[\text{(electron-hole representation)}]{\text{Nambu space}}$ $H_{pair} = \sum_{k} \Delta_s d^{\dagger}_{\vec{k},\uparrow} C_{\vec{k},\downarrow} + h.c.$ $d_{\vec{k},\uparrow} = C^{\dagger}_{-\vec{k},\uparrow}$ hole operator

excitation in s-wave SC. $b_{\vec{k}} = u C_{\vec{k},\uparrow} + v d_{\vec{k},\downarrow} = \underbrace{u C_{\vec{k},\uparrow} + v C^{\dagger}_{-\vec{k},\downarrow}}$. not self-conjugate

⇒ SC formed with identical spin

odd parity

p-wave SC. $H^{p}_{pair} = \sum_{p} \Delta_p(\vec{k}) C_{\vec{k}} C_{-\vec{k}} + h.c.$ $\Delta_p(\vec{k}) = -\Delta_p(-\vec{k})$.

excitation $\gamma_{\vec{k}} = u C_{\vec{k}} + v C_{\vec{k}}$. $\xrightarrow{\text{real space}}$ $\gamma(\vec{x}) = u(\vec{x}) c(\vec{x}) + v(\vec{x}) c^{\dagger}(\vec{x})$. $u = v^*$. majorana.

MZM. in 1D p-wave TSC. boundary. 2D p+ip TSC. vortice center.

self-conjugate. do not have number space. $\gamma^{\dagger}\gamma = \gamma^2 = 1$.

2 MZM → Dirac fermion. $|0\rangle . |1\rangle$. MZM. quantum dimension $\sqrt{2}$. half complex fermion.

non-Abelian statistics

QI stored in a non-local form

1D spinless p-wave SC chain

$\{c_i, c_j\} = \{c_i^+, c_j^+\} = 0 \quad i \neq j$

$\{c_i, c_j^+\} = \delta_{ij}$.

$$H = -\mu \sum_{k=1}^{N} c_k^+ c_k - \sum_{k=1}^{N-1} (t c_k^+ c_k + \underbrace{\Delta}_{\text{nearest-neighbour pairing strength}} e^{i\overset{\text{SC pairing phase}}{\phi}} c_k c_k + h.c.)$$

$c_i c_j + c_j c_i = 0$

$\gamma_{k,A} = i(c_k^+ e^{-i\phi/2} - c_k e^{i\phi/2})$.

$\gamma_{k,B} = c_k^+ e^{-i\phi/2} + c_k e^{i\phi/2}$.

$\{\gamma_{\alpha,k}, \gamma_{\alpha',k'}\} = 2\delta_{\alpha\alpha'}\delta_{kk'}$.

$c_i c_j^+ + c_j^+ c_i = 0. \ (i \neq j)$

$c_i c_i^+ + c_i^+ c_i = 1$.

$$H = -\frac{\mu}{2} \sum_{k=1}^{N} (1 + i\gamma_{k,B}\gamma_{k,A}) - \frac{i}{4} \sum_{x-1}^{N-1} \left[(\Delta+t)\gamma_{k,B}\gamma_{k+1,A} + (\Delta-t)\gamma_{k,A}\gamma_{k+1,B} \right].$$

$c_k e^{i\phi/2} = (\gamma_{k,B} + i\gamma_{k,A})/2$

$\gamma_{k,A}^2 + \gamma_{k,B}^2 = 2(c_k^+ c_k + c_k c_k^+)$

$c_k^+ e^{-i\phi/2} = (\gamma_{k,B} - i\gamma_{k,A})/2$

$(\gamma_{k,B} - i\gamma_{k,A})(\gamma_{k,B} + i\gamma_{k,A})$

$c_k^+ c_k = \gamma_{k,A}^2 + \gamma_{k,B}^2 - i\gamma_{k,A}\gamma_{k,B} + i\gamma_{k,B}\gamma_{k,A}$

$\{\gamma_{k,A}, \gamma_{k,B}\} = \gamma_{k,A}\gamma_{k,B} + \gamma_{k,B}\gamma_{k,A} = 0$

$i\gamma_{k,B}\gamma_{k,A} = -1(c_k^+ c_k^+ e^{-i\phi} - c_k^+ c_k + c_k c_k^+ - c_k c_k e^{i\phi})$

$= i(c_k^+ c_k^+ e^{-i\phi} - c_k c_k^+ + c_k^+ c_k - c_k c_k e^{i\phi})$

$+ i(c_k^+ c_k^+ e^{-i\phi} + c_k c_k^+ - c_k^+ c_k - c_k c_k e^{i\phi})$

?

$= 2i(c_k^+ c_k^+ e^{-i\phi} - c_k c_k e^{i\phi})$?

$$H_1 = \sum_j \left[-t(\underbrace{a_j^+ a_{j+1} + a_{j+1}^+ a_j}_{\neq 0 \text{ for periodic conditions}}) - \mu(a_j^+ a_j - \tfrac{1}{2}) + \Delta a_j a_{j+1} + \Delta^* a_{j+1}^+ a_j^+ \right].$$

$\Delta = |\Delta| e^{i\theta}$.

$c_{2j-1} = e^{i\frac{\theta}{2}} a_j + e^{-i\frac{\theta}{2}} a_j^+$

$a_j = (c_{2j-1} + i c_{2j})/2 \cdot e^{-i\frac{\theta}{2}}$

$c_{2j} = -i(e^{i\frac{\theta}{2}} a_j - e^{-i\frac{\theta}{2}} a_j^+)$.

$a_j^+ = (c_{2j-1} - i c_{2j})/2 \cdot e^{i\frac{\theta}{2}}$

$$a_j^+ a_{j+1} + a_{j+1}^+ a_j = \frac{c_{2j-1} - i c_{2j}}{2} \cdot \frac{c_{2j+1} + i c_{2j+2}}{2} + \frac{c_{2j+1} - i c_{2j+2}}{2} \cdot \frac{c_{2j-1} + i c_{2j}}{2}$$

$c_{2j-1}^+ = e^{-\frac{i\theta}{2}} a_j^+ + e^{i\frac{\theta}{2}} a_j = c_{2j-1}$. ✓

$c_{2j-1}^2 = c_{2j-1}^{+2} = 1$?

$\{c_i, c_j\} = 2\delta_{ij}$?

$c_i c_j + c_j c_i = 0$

$c_j^+ = c_j \rightarrow c_j^2 = c_j^+ c_j = c_j c_j^+ = 1$?

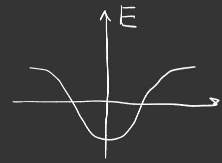
$$= \Big(\underbrace{c_{2j-1} c_{2j+1}}_{\parallel 0} - i c_{2j} c_{2j+1} + i c_{2j-1} c_{2j+2} + \underbrace{c_{2j} c_{2j+2}}_{\parallel 0}$$

$$+ c_{2j+1} c_{2j-1} - i c_{2j+2} c_{2j-1} + i c_{2j+1} c_{2j} + c_{2j+2} c_{2j} \Big) \cdot \frac{1}{4}$$

$$= \frac{i}{2}\left(c_{2j-1}c_{2j+2} + c_{2j+1}c_{2j}\right)$$

推一遍?

$$\cdots \quad H_1 = \frac{i}{2}\sum_j \left(-\mu c_{2j-1}c_{2j} + (\omega+|\Delta|)\, c_{2j}c_{2j+1} + (-\omega+|\Delta|)\, c_{2j-1}c_{2j+2}\right).$$



$\mu \neq 0.\ \Delta = t = 0.\quad H \cdots \Longrightarrow -\mu \sum_j c_j^\dagger c_j.\ \ a_j a_j$

couple. $\Rightarrow$ fermionic excitation

no MZM. no zero-energy excitation.

$\mu = 0.\ \Delta = t.$  couple through Δ, t.

$\gamma_{1,A}, \gamma_{N,B}$ not shown in Hamiltonian!

MZM on two ends.

$f = \frac{1}{2}(\gamma_{1,A} + i\gamma_{N,B})$. zero excitation energy?

why? 2-fold degenerate ground state $|0\rangle$. $f^\dagger|0\rangle = |1\rangle$.

$$H = \frac{1}{2}\sum_k C_k^\dagger H(k) C_k. \qquad C_k = (c_k, c_{-k}^\dagger).\quad \text{Nambu space}$$

真的是完全不同的拓扑相吗?

$$H(k) = \begin{pmatrix} -t\cos k - \mu & i\Delta e^{-i\phi}\sin k \\ -i\Delta e^{i\phi}\sin k & t\cos k + \mu \end{pmatrix}.$$

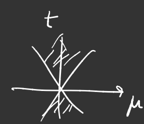
topological invariant. 1D winding number?

k change 2π.

$\phi = 0.\quad H(k) = h_z \tau_z + h_y \tau_y.$

$\vec{h}(k) = (h_y, h_z)$ wind in y-z plane.

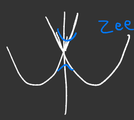
for Kitaev chain. $N = [\mathrm{sgn}(h_z(0)) - \mathrm{sgn}(h_z(\pi))]/2.\quad h_z = -\mu - t\cos k.$



必须关闭能隙? $|\mu| < t$.

MZM. coupled energy $E_f \propto e^{-L}$.

1D TSC. SOC. s-wave. Zeeman field. induce equivalent spinless p-wave SC.



non-Abelian statistics

U(1) phase many-body system. degenerate ground state.

braiding. adiabatic. $H \longrightarrow H_0$.

$$|\psi_f(t)\rangle = \underbrace{e^{-\frac{i}{\hbar}\int dt\, E(t)}}_{\text{dynamical phase}} \underbrace{\vec{B}_0(t)}_{\text{Berry matrix}} |\alpha(t)\rangle.$$

$|\phi_f\rangle = \underbrace{U_{fi}}_{\text{matrix in degenerate space}} |\phi_i\rangle$. do not commute. non-Abelian

2N. seperate far MZM.

$$f_i = \frac{\gamma_{2i-1} + i\gamma_{2i}}{2}. \qquad \varepsilon_i = 0.$$

$$|\phi_i\rangle = |n_1 \cdots n_N\rangle.$$

$$i\hbar \frac{d\psi}{dt} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi . \qquad \frac{d\psi}{dt} = \left(-i\hbar c\vec{\alpha}\,\nabla + \beta mc^2\right)\psi .$$

Solution: $(i\hbar\gamma^\mu \partial_\mu - mc)\Psi = 0$.

$c_i^\dagger$. generate an electron. (annihlate a hole)

$$c_i^2 = (c_i^\dagger)^2 = 0 . \qquad \{c_i^\dagger, c_i\} = 1 .$$

$$\{c_i^\dagger, c_j\} = c_i^\dagger c_j + c_j c_i^\dagger = \{c_i, c_j\} = \{c_i^\dagger, c_j^\dagger\} = 0$$

$$\gamma_{i,1}^2 = \gamma_{i,2}^2 = 1 .$$

$$c_i = \frac{1}{2}(\gamma_{i,1} + i\gamma_{i,2}) \qquad \gamma_{i,1} = c_i^\dagger + c_i . \qquad \gamma_{i,1}\gamma_{i,2} + \gamma_{i,2}\gamma_{i,1} = 2$$

$$c_i^\dagger = \frac{1}{2}(\gamma_{i,1} - i\gamma_{i,2}) . \qquad \gamma_{i,2} = i(c_i^\dagger - c_i) . \qquad \gamma_{i,1}\gamma_{j,2} + \gamma_{j,2}\gamma_{i,1} = 0 .$$

$$H = -\mu \sum_i^N c_i^\dagger c_i - \sum_i^{N-1}\left(t c_i^\dagger c_{i+1} + \Delta c_i c_{i+1} + h.c.\right)$$

$$c_i^\dagger c_i = \frac{1}{4}\left(\gamma_{i,1}^2 - i\gamma_{i,2}\gamma_{i,1} + i\gamma_{i,1}\gamma_{i,2} + \gamma_{i,2}^2\right) .$$

$$c_i^\dagger c_{i+1} =$$

$$\tilde{c}_i = \frac{\gamma_{i+1,1} + i\gamma_{i,2}}{2} .$$

$$\Rightarrow H = -it \sum_{i=1}^{N-1} \gamma_{i,2}\gamma_{i+1,1} = 2t \sum_{i=1}^{N-1} \tilde{c}_i^\dagger \tilde{c}_i .$$

0407088

type-II quantum computing.

utilize coherent charge transfer in an incoherent device

i.f states are eigenstates of H. intermediate states formed by coherent superposition

enough coherence to effect SWAP gate. not enough coherence to maintain any superpositions after the initial transfer complete

coherent population transfer — non-adiabatic

(adiabatic. ⟷ adiabatic passage

quasi-static population transfer

$\Downarrow$

simultaneous modulation of at least two system parameters to realize a desired trajectory through the Hilbert space

control of tunneling to realize π or similar pulses

SNR. pulse switching time. X.

EM mediated adiabatic passage in quantum computing?

generating entanglement via adiabatic passage?

maintaining constant energy. prevent dynamical phase when QI transported.

donors. $|1\rangle$, $|2\rangle$, $|3\rangle$. (position eigenstates) shift gates. barrier gates.

usually gaussian pulses?

$$c = \gamma(x) + \gamma^\dagger(x)$$

$$c^\dagger = -i\left(\gamma(x) - \gamma^\dagger(x)\right)$$

$$ic^\dagger = \gamma(x) - \gamma^\dagger(x)$$

张量积 $\otimes$ 意味着系统的某种局域性?

简并压?

1D TFIM $\quad H = -J\sum_{i=1}^{L} \sigma_i^x \sigma_{i+1}^x - h\sum_{i=1}^{L} \sigma_i^z$.

$S_j^+ = c_j^+ e^{i\pi \sum_{k<j} c_k^+ c_k}$ JW string $\quad [S_j^+, S_k^-]$ for $j=k$, $j<k$.

$S_j^- = e^{-i\pi \sum_{k<j} c_k^+ c_k} c_j$

$S_j^z = c_j^+ c_j - \frac{1}{2}$.

$$[S_j^+, S_k^-] = c_j^+ e^{i\pi \sum_{l<j} c_l^+ c_l} e^{-i\pi \sum_{m<k} c_m^+ c_m} c_k - e^{-i\pi \sum_{m<k} c_m^+ c_m} c_k c_j^+ e^{i\pi \sum_{l<j} c_l^+ c_l}.$$

① $j=k$. $\quad [S_j^+, S_k^-] = c_j^+ e^{i\pi \sum_{l<j} c_l^+ c_l} e^{-i\pi \sum_{l<j} c_l^+ c_l} c_j - e^{-i\pi \sum_{l<j} c_l^+ c_l} c_j c_j^+ e^{i\pi \sum_{l<j} c_l^+ c_l}$

$e^{i\pi (c_1^+ c_1 + c_2^+ c_2 + \cdots c_j^+ c_j)}$.

$c_l^+ c_l + c_l c_l^+ = 1$. $\qquad = c_j^+ c_j - c_j c_j^+ = 2n_j - 1$

$c_j^+ c_j + c_j c_j^+ = 1$

n_j occupation number on jth site

$n_j - \frac{1}{2} = S_j^z$? ✓

$-\frac{1}{2}$ or $\frac{1}{2}$. $\longrightarrow$ 0 or 1.

② $j<k$. $\quad [S_j^+, S_k^-] = c_j^+ e^{i\pi \sum_{l<j} c_l^+ c_l} e^{-i\pi \sum_{m<k} c_m^+ c_m} c_k$

? $\qquad - e^{-i\pi \sum_{m<k} c_m^+ c_m} c_k c_j^+ e^{i\pi \sum_{l<j} c_l^+ c_l}$.

$= c_j c_k^+ e^{-i\pi \sum_{l<j} c_l^+ c_l} e^{-i\pi \sum_{m<k} c_m^+ c_m}$

$- c_k c_j^+ e^{-i\pi \sum_{m<k} c_m^+ c_m} e^{i\pi \sum_{l<j} c_l^+ c_l}$

$= \{c_j, c_k^+\} \cdots = 0$.

$$e^{-i\pi c_j^\dagger c_j} = 1 - 2c_j^\dagger c_j .$$

$$c_j^\dagger c_j = n_j \in \{0,1\}$$

how to prove?

$$e^{-i\pi c_j^\dagger c_j} = \sum_{n=0}^{\infty} \frac{(-i\pi c_j^\dagger c_j)^n}{n!} .$$

$$= 1 - i\pi c_j^\dagger c_j + \frac{(-i\pi)^2}{2}(c_j^\dagger c_j)^2 + \cdots$$

$$(c_j^\dagger c_j)^2 = c_j^\dagger c_j c_j^\dagger c_j$$

$$= c_j^\dagger (1 - c_j c_j^\dagger) c_j = c_j^\dagger c_j - c_j^\dagger c_j c_j^\dagger c_j$$

$$\Rightarrow \left[e^{-i\pi c_j^\dagger c_j}, c_j^\dagger\right] = (1 - 2c_j^\dagger c_j)c_j^\dagger - c_j^\dagger(1 - 2c_j^\dagger c_j)$$

$$= -2 c_j^\dagger c_j c_j^\dagger = -2c_j^\dagger(1 - 2c_j^\dagger c_j) = -2c_j^\dagger e^{-i\pi c_j^\dagger c_j} .$$

$$S_x = \tfrac{1}{2}\sigma_x . \qquad S^+ = S_x + iS_y .$$

$$S^- = S_x - iS_y .$$

$$\sigma_x = S^+ + S^- .$$

$$c_i c_i^\dagger - \frac{1}{2} = \frac{1}{2}(c_i^\dagger c_i + c_i c_i^\dagger)$$

$$H = -J\sum_{i=1}^{L} \sigma_i^x \sigma_{i+1}^x - h\sum_{i=1}^{L}\sigma_i^z$$

$$= -J\sum_{i=1}^{L}(S_i^+ + S_i^-)(S_{i+1}^+ + S_{i+1}^-) - h\sum_{i=1}^{L}(\cancel{2c_i^\dagger c_i} - 1)$$

$$c_i^\dagger c_i + c_i c_i^\dagger = 2c_i^\dagger c_i + \{c_i, c_i^\dagger\} = 2c_i^\dagger c_i + 1 .$$

$$= 2c_i c_i^\dagger - 1 .$$

$$\sigma_i^z = 2S_i^z = 2(c_i^\dagger c_i - \tfrac{1}{2})$$

$$1 - 2c_j^\dagger c_j ?$$

$$H = -\mu \sum_i c_i^\dagger c_i - \frac{1}{2}\sum_i \left(t c_i^\dagger c_{i+1} + \Delta c_i c_{i+1} + h.c. \right)$$

$$c_j = \frac{1}{\sqrt{N}} \sum_k e^{ikj} c_k \, . \qquad c_j^\dagger = \frac{1}{\sqrt{N}} \sum_k e^{-ikj} c_k^\dagger \, .$$

$$\Longrightarrow \quad H = -\mu \sum_k c_k^\dagger c_k - \sum_k \left(t\cos k \, c_k^\dagger c_k - \Delta c_k^\dagger c_k^\dagger e^{ik} + c_k c_k e^{-ik} \right)$$

$$H = -\mu \sum_i \left(c_i^\dagger c_i - \frac{1}{2} \right) - \sum_i \left(t c_i^\dagger c_{i+1} + \Delta c_i c_{i+1} + h.c. \right) .$$

$$\begin{cases} c_i = \dfrac{1}{\sqrt{L}} \sum_k e^{ikx_i} c_k \\ c_i^\dagger = \dfrac{1}{\sqrt{L}} \sum_k e^{-ikx_i} c_k^\dagger \end{cases}$$

periodic condition

$$\Rightarrow \quad H = -\mu \sum_{i,k} \frac{1}{L} c_k^\dagger c_k - \sum_{i,k} \left(\underbrace{\frac{t}{L} e^{ik(x_{i+1}-x_i)}}_{t_k} c_k^\dagger c_k + \underbrace{\frac{\Delta}{L} e^{ik(x_{i+1}+x_i)}}_{\Delta_k} c_k c_k + h.c. \right)$$

$$+ \frac{1}{2}\mu L$$

$$= -\mu \sum_k \left(c_k^\dagger c_k - \frac{1}{2} \right) - \sum_k \left(t_k c_k^\dagger c_k + \Delta_k c_k c_k + h.c. \right)$$

$$t\left(c_1^\dagger c_2 + c_2^\dagger c_3 + \cdots \, c_{L-1}^\dagger c_L + c_L^\dagger c_1 \right)$$

$$\frac{1}{\sqrt{L}} \sum_k e^{-ik} c_k^\dagger \cdot \frac{1}{\sqrt{L}} \sum_k e^{2ik} c_k + \cdots$$

$$\frac{1}{L} \sum_k e^{ik} c_k^\dagger c_k + \cdots$$

$$t_k = \sum_i e^{ik}/L = e^{ik}.$$

$$\sum_{j=0}^{L-1} r^j = \frac{1-r^L}{1-r}$$

$$\Delta_k = \sum_i e^{ik(x_i + x_{j+1})}$$

$$\sum_k e^{ikx} = e^{ix} + e^{2ix} + \cdots e^{iLx}$$

$$r = e^{ix}. \qquad \text{Sum} = \frac{1-e^{iLx}}{1-e^{ix}}$$

$x = 0.$ $\quad$ sum $= 1.$

$x \neq 0.$ $\quad$ sum $=$

$$\sum_{kl} \frac{1}{L} e^{\frac{2\pi i}{L}(kj - l(j+1))} \; C_k^+ C_l$$

$$\sum_{kl} \frac{1}{L} e^{\frac{2\pi i}{L}(k-l)j} \quad e^{-\frac{2\pi i l}{L}}$$

$$\frac{1}{N}\sum_{n=0}^{N-1} e^{i(k-l)\frac{2\pi}{N}n} = \begin{cases} 1. & k=l \\ 0. & k \neq l. \end{cases}$$

orthogonal

$k = l.$ $\quad \sum_k \frac{1}{L} = 1.$

$k \neq l.$ $\quad \frac{1}{L} e^{\frac{2\pi i}{L} jn} = 0\,?$

$$-kj + lj + l$$

$$\sum_l \frac{1}{L} e^{-\frac{2\pi i k}{L}} C_k^+ C_k$$

$$j(l-k) + l$$

$$e^{-ik} C_k^+ C_k$$

easier approach ?

$$\sum_{l} e^{i(k-l)\cdot\frac{2\pi}{L}} c_k^\dagger c_l = \delta_{kx} - \sum_{l} e^{i(k-l)\cdot\frac{2\pi}{L}} c_l c_k^\dagger .$$

$$c_l c_k^\dagger = \frac{1}{L}\sum_{p,q} e^{i\left(p\frac{2\pi x}{L} - q\frac{2\pi y}{L}\right)} c_p c_q^\dagger .$$

$$\mu\left(c_k^\dagger c_k - \frac{1}{2}\right) + \sum t_k c_k^\dagger c_k + t_k^\dagger c_k c_k^\dagger + \Delta_k c_k^\dagger c_{-k}^\dagger + \Delta_k^* c_k c_{-k}$$

$$(t_k + t_k^\dagger)\, c_k^\dagger c_k + t_k^\dagger\left(c_k c_k^\dagger - c_k^\dagger c_k\right)$$

$$+ (\Delta_k - \Delta_k^*)\, c_k^\dagger c_{-k}^\dagger + \Delta_k^*\left(c_k c_{-k} + c_k^\dagger c_{-k}^\dagger\right)$$

$$\begin{pmatrix} c_k^\dagger & c_{-k} \end{pmatrix} \begin{pmatrix} c_k \\ c_{-k}^\dagger \end{pmatrix}$$

$$c_k^\dagger c_k \qquad c_{-k} c_k$$

$$c_k^\dagger c_{-k}^\dagger \qquad c_k c_{-k}^\dagger$$

$$\underline{c_i c_{i+1}} + c_i^\dagger c_{i+1} + c_i c_{i+1}^\dagger + \underline{c_i^\dagger c_{i+1}^\dagger}$$

$$- J\sum (c_i + c_i^\dagger)(c_{i+1} + c_{i+1}^\dagger) - h\sum (1 - 2c_i^\dagger c_i)$$

$$c_i c_i^\dagger + c_i^\dagger c_i$$

$$2h\sum_i \left(c_i^\dagger c_i - \frac{1}{2}\right) + \sum_i J\left(c_i c_{i+1} + c_i^\dagger c_{i+1} + \underline{c_i c_{i+1}^\dagger} + \underline{c_i^\dagger c_{i+1}^\dagger}\right)$$

$$- c_{i+1}^\dagger c_i \qquad - c_{i+1}^\dagger c_i^\dagger$$

$$H = -\sum_{k=1}^{L} \mu\left(c_k c_k^\dagger - \frac{1}{2}\right) + t e^{ik} c_k^\dagger c_k + t e^{ik} c_k c_k^\dagger$$

$$c_k^\dagger c_k$$

$$\sum_k (\mu + t_k)\, c_k^\dagger c_k - \frac{\mu L}{2} + \sum_k t_k^* c_k c_k^\dagger$$

$$+ \sum_k \Delta_k \left(c_k^\dagger c_{-k}^\dagger + c_k c_{-k}\right)$$

$$c_k^\dagger c_k + c_k c_k^\dagger = 1.$$

$$(2t\cos k + \mu)\, c_k^\dagger c_k - \frac{\mu L}{2} + \Delta e^{ik}\left(c_k^\dagger c_k + c_k c_k\right)$$

$$\Delta_k c_k^\dagger c_k^\dagger + \Delta_k^* c_k c_k \qquad c_k^\dagger c_k \quad c_k^\dagger c_k$$

$$c_k c_k^\dagger$$

$$-\frac{J}{4}\sum (c_i + c_i^{+})(c_{i+1} + c_{i+1}^{+})$$

Kitaev Hamiltonian in momentum space

$$H = -\mu \sum_i \left(c_i^\dagger c_i - \frac{1}{2}\right) - \sum_i \left(t\, c_i^\dagger c_{i+1} + \Delta c_i c_{i+1} + h.c.\right)$$

From trasverse field Ishg Hamiltonian to Kitaev

包含单体、多体相互作用的 Hamiltonian

$$\hat{H} = \sum_{ij} \varepsilon_{ij} \hat{C}_i^{+} \hat{C}_j + \sum_{ij,m,n} V_{ijmn} \hat{C}_i^{+} \hat{C}_j^{+} \hat{C}_m \hat{C}_n + \cdots$$

若有幺正变换将其对角化

$$\hat{H} = \sum_i E_i \hat{C}_i^{+} \hat{C}_i \,.$$ $\longrightarrow$ 元激发、能谱 E_i.